State whether each quantity described is a *vector* quantity or a *scalar* quantity.

11. a car driving 50 miles an hour due east

SOLUTION:

This quantity has a magnitude of 50 miles an hour and a direction of due east. This is a vector quantity.

12. a gust of wind blowing 5 meters per second

SOLUTION:

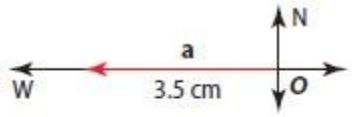
This quantity has a magnitude of 5 meters per second but no direction. This is a scalar quantity.

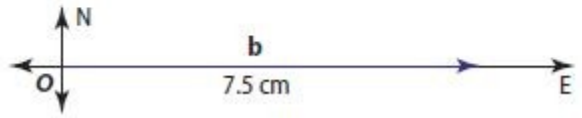
Determine the magnitude and direction of the resultant of each vector sum.

17. 70 meters due west and then 150 meters due east

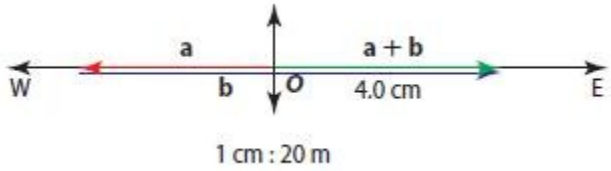
SOLUTION:

Let **a** = 70 meters due west and **b** = 150 meters due east. Draw a diagram to represent **a** and **b** using a scale of 1 cm : 20 m.





Translate **b** so that its tail touches the tip of **a**. Then draw the resultant vector **a + b**.



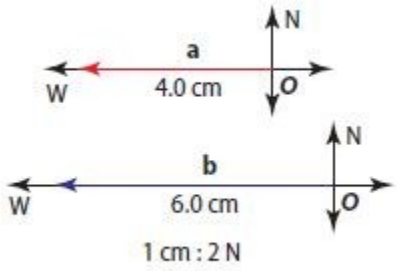
Drawings may not be to scale.

Measure the length of **a + b**. The length of the vector is approximately 4.0 centimeters, which is 4.0 × 20 or 80 meters. **a + b** is in the direction of **b**. Since the direction of **b** is due east, the resultant vector is 80 meters due east.

18. 8 newtons directly backward and then 12 newtons directly backward

SOLUTION:

Let **a** = 8 newtons directly backward and **b** = 12 newtons directly backward. Draw a diagram to represent **a** and **b** using a scale of 1 cm : 2 N. Let due west be backward.



Translate **b** so that its tail touches the tip of **a**. Then draw the resultant vector **a + b**.

Drawings may not be to scale.

Measure the length of **a + b**. The length of the vector is approximately 10.0 centimeters, which is 10.0 × 2 or 20 newtons. **a + b** is in the direction of **a** and **b**. Since the direction of both vectors is due west, the resultant vector is 20 newtons backward.

Find the component form and magnitude of $\overrightarrow{AB}$ with the given initial and terminal points.

19. $A(-1, 3)$, $B(5, 4)$

SOLUTION:

First, find the component form.

$$\begin{aligned}\overrightarrow{AB} &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle 5-(-1), 4-3 \rangle \\ &= \langle 6, 1 \rangle\end{aligned}$$

Next, find the magnitude. Substitute $x_2 - x_1 = 6$ and $y_2 - y_1 = 1$ into the formula for the magnitude of a vector in the coordinate plane.

$$\begin{aligned}\left|\overrightarrow{AB}\right| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{6^2 + 1^2} \\ &= \sqrt{37} \text{ or about } 6.1\end{aligned}$$

20. $A(7, -2)$, $B(-9, 6)$

SOLUTION:

First, find the component form.

$$\begin{aligned}\overrightarrow{AB} &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle -9 - 7, 6 - (-2) \rangle \\ &= \langle -16, 8 \rangle\end{aligned}$$

Next, find the magnitude. Substitute $x_2 - x_1$ = -16 and $y_2 - y_1$ = 8 into the formula for the magnitude of a vector in the coordinate plane.

$$\begin{aligned}\left|\overrightarrow{AB}\right| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-16)^2 + 8^2} \\ &= \sqrt{320} \\ &= 8\sqrt{5} \text{ or about } 17.9\end{aligned}$$

Find each of the following for p = $\langle 4, 0 \rangle$, q = $\langle -2, -3 \rangle$, and t =$\langle -4, 2 \rangle$.

26. 2**p** + **t** – 3**q**

SOLUTION:

$$\begin{aligned}\mathbf{2p} + \mathbf{t} - 3\mathbf{q} &= \mathbf{2p} + \mathbf{t} + (-3)\mathbf{q} \\ &= (2)\langle 4, 0 \rangle + \langle -4, 2 \rangle + (-3)\langle -2, -3 \rangle \\ &= \langle 8, 0 \rangle + \langle -4, 2 \rangle + \langle 6, 9 \rangle \\ &= \langle 10, 11 \rangle\end{aligned}$$

Find a unit vector u with the same direction as v.

27. $\mathbf{v} = \langle -7, 2\rangle$

SOLUTION:

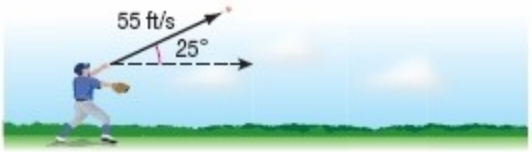
$$\begin{aligned}\mathbf{u} &= \frac{1}{|\mathbf{v}|}\mathbf{v}\\ &= \frac{1}{|\langle -7,2\rangle|}\langle -7,2\rangle\\ &= \frac{1}{\sqrt{(-7)^2+2^2}}\langle -7,2\rangle\\ &= \frac{1}{\sqrt{53}}\langle -7,2\rangle\\ &= \left\langle -\frac{7}{\sqrt{53}}, \frac{2}{\sqrt{53}}\right\rangle\\ &= \left\langle -\frac{7\sqrt{53}}{53}, \frac{2\sqrt{53}}{53}\right\rangle\end{aligned}$$

28. $\mathbf{v} = \langle 3, -3\rangle$

SOLUTION:

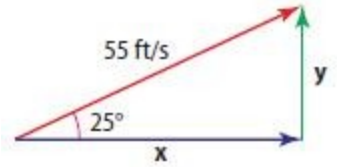
$$\begin{aligned}\mathbf{u} &= \frac{1}{|\mathbf{v}|}\mathbf{v}\\ &= \frac{1}{|\langle 3, -3\rangle|}\langle 3, -3\rangle\\ &= \frac{1}{\sqrt{3^2+(-3)^2}}\langle 3, -3\rangle\\ &= \frac{1}{\sqrt{18}}\langle 3, -3\rangle\\ &= \frac{1}{3\sqrt{2}}\langle 3, -3\rangle\\ &= \left\langle \frac{3}{3\sqrt{2}}, -\frac{3}{3\sqrt{2}}\right\rangle\\ &= \left\langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right\rangle\end{aligned}$$

53. **BASEBALL** A player throws a baseball with an initial velocity of 55 feet per second at an angle of 25° above the horizontal, as shown below. Find the magnitude of the horizontal and vertical components.



SOLUTION:

The horizontal and vertical components of the vector form a right triangle. Use the sine or cosine ratios to find the magnitude of each component.



$$\cos 25^\circ = \frac{|\mathbf{x}|}{55} \qquad \sin 25^\circ = \frac{|\mathbf{y}|}{55}$$

$$|\mathbf{x}| = 55\cos 25^\circ \qquad |\mathbf{y}| = 55\sin 25^\circ$$

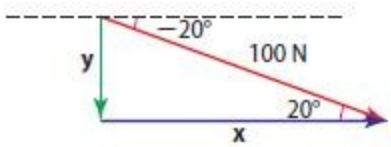
$$|\mathbf{x}| \approx 49.8 \qquad |\mathbf{y}| \approx 23.2$$

The magnitude of the horizontal component is about 49.8 feet per second and the magnitude of the vertical component is about 23.2 feet per second.

54. **STROLLER** Miriam is pushing a stroller with a force of 200 newtons at an angle of 20° below the horizontal. Find the magnitude of the horizontal and vertical components of the force.

SOLUTION:

Diagram the situation.



The horizontal and vertical components of the vector form a right triangle. Use the sine or cosine ratios to find the magnitude of each component.

$$\cos 20^\circ = \frac{|\mathbf{x}|}{200} \qquad \sin 20^\circ = \frac{|\mathbf{y}|}{200}$$

$$|\mathbf{x}| = 200\cos 20^\circ \qquad |\mathbf{y}| = 200\sin 20^\circ$$

$$|\mathbf{x}| \approx 187.9 \qquad |\mathbf{y}| \approx 68.4$$

The magnitude of the horizontal component is about 187.9 newtons and the magnitude of the vertical component is about 68.4 newtons.