Mid-Chapter Quiz: Lessons 7-1 through 7-3

Graph the hyperbola given by each equation.

13. $\frac{x^2}{81} - \frac{(y+7)^2}{81} = 1$

SOLUTION:

The equation is in standard form, with $h = 0$ and $k = -7$. Because $a^2 = 81$ and $b^2 = 81$, $a = 9$ and $b = 9$. The values of a and b can be used to find c.

$c^2 = a^2 + b^2$

$c^2 = 81 + 81$

$c = \sqrt{162}$ or about 12.73

Use h, k, a, b, and c to determine the characteristics of the hyperbola.

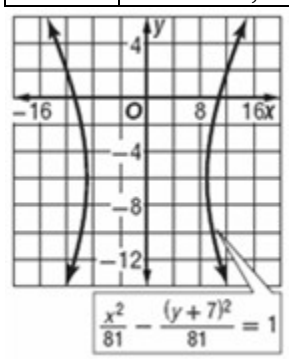
orientation: In the standard form of the equation, the y–term is being subtracted. Therefore, the orientation of the hyperbola is horizontal.

center: $(h, k) = (0, -7)$
vertices: $(h \pm a, k) = (-9, -7)$ and $(9, -7)$
foci: $(h \pm c, k) = (-12.73, -7)$ and $(12.73, -7)$
asymptotes:

$$\begin{aligned} y - k &= \pm \tfrac{b}{a}(x - h) \\ y - (-7) &= \pm \tfrac{9}{9}(x - 0) \\ y + 7 &= \pm 1x \\ y &= \pm [x] - 7 \\ y &= x - 7 \quad OR \\ y &= -x - 7 \end{aligned}$$

Graph the center, vertices, foci, and asymptotes. Then make a table of values to sketch the hyperbola.

x	y
–15	–19, 5
–11	–13.3, –0.68
11	–13.3, –0.68
15	–19, 5



14. $\frac{(y-3)^2}{4} - \frac{(x-3)^2}{16} = 1$

SOLUTION:

The equation is in standard form, with $h = 3$ and $k = 3$. Because $a^2 = 4$ and $b^2 = 16$, $a = 2$ and $b = 4$. The values of a and b can be used to find c.

$c^2 = a^2 + b^2$

$c^2 = 4 + 16$

$c = \sqrt{20}$ or about 4.47

Use h, k, a, b, and c to determine the characteristics of the hyperbola.

orientation: In the standard form of the equation, the x–term is being subtracted. Therefore, the orientation of the hyperbola is vertical.
center: $(h, k) = (3, 3)$
vertices: $(h, k \pm a) = (3, 1)$ and $(3, 5)$
foci: $(h, k \pm c) = (3, -1.47)$ and $(3, 7.47)$
asymptotes:

$$y - k = \pm\frac{a}{b}(x - h)$$

$$y - 3 = \pm\frac{2}{4}(x - 3)$$

$$y - 3 = \pm\frac{1}{2}(x - 3)$$

$$y - 3 = \pm\left[\frac{1}{2}x - \frac{3}{2}\right]$$

$$y = \pm\left[\frac{1}{2}x - \frac{3}{2}\right] + 3$$

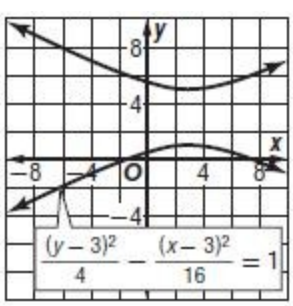
$$y = \pm\left[\frac{1}{2}x - \frac{3}{2}\right] - \frac{6}{2}$$

$$y = \frac{1}{2}x - \frac{9}{2} \quad OR$$

$$y = -\frac{1}{2}x - \frac{3}{2}$$

Graph the center, vertices, foci, and asymptotes. Then make a table of values to sketch the hyperbola.

x	y
0	0.5, 5.5
2	0.94, 5.06
4	0.94, 5.06
6	0.5, 5.5



Write an equation for the hyperbola with the given characteristics.

15. vertices (0, 5), (0, –5); conjugate axis length of 6

SOLUTION:

Because the x–coordinates of the vertices are the same, the transverse axis is vertical, and the standard form of the equation is $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$.

The center is located at the midpoint of the vertices, or (0, 0). So, $h = 0$ and $k = 0$. Because the vertices are 10 units apart, $2a = 10$, $a = 5$, and $a^2 = 25$. The length of the conjugate axis is 6 units, so $2b = 6$, $b = 3$, and $b^2 = 9$.

Using the values of h, k, a, and b, the equation for the hyperbola is $\frac{y^2}{25} - \frac{x^2}{9} = 1$.

16. foci (10, 0), (–6, 0); transverse axis length of 4

SOLUTION:

Because the y–coordinates of the foci are the same, the transverse axis is horizontal, and the standard form of the equation is $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$.

The center is the midpoint of the segment between the foci, or (2, 0). So, $h = 2$ and $k = 0$. The length of the transverse axis of a hyperbola is $2a$. So, $2a = 4$, $a = 2$, and $a^2 = 4$. You can find c by determining the distance from a focus to the center. One focus is located at (10, 0), which is 8 units from (2, 0). So, $c = 8$.

Now you can use the values of a and c to find b.

$$\begin{aligned} b^2 &= c^2 - a^2 \\ b^2 &= 8^2 - 2^2 \\ b^2 &= 64 - 4 \\ b^2 &= 60 \\ b &= \sqrt{60} \end{aligned}$$

Using the values of h, k, a, and b, the equation for the hyperbola is $\frac{(x-2)^2}{4} - \frac{y^2}{60} = 1$.

17. vertices (–11, 0), (11, 0); foci (–14, 0), (14, 0)

SOLUTION:

Because the y–coordinates of the vertices are the same, the transverse axis is horizontal, and the standard form of the equation is $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$.

The center is the midpoint of the segment between the foci, or (0, 0). So, $h = 0$ and $k = 0$. You can find c by determining the distance from a focus to the center. One focus is located at (14, 0), which is 14 units from (0, 0). So, $c = 14$. You can find a by determining the distance from a vertex to the center. One vertex is located at (11, 0), which is 11 units from (0, 0). So, $a = 11$ and $a^2 = 121$.

Now you can use the values of c and a to find b.

$$\begin{aligned} b^2 &= c^2 - a^2 \\ b^2 &= 14^2 - 11^2 \\ b^2 &= 196 - 121 \\ b^2 &= 75 \\ b &= \sqrt{75} \end{aligned}$$

Using the values of h, k, a, and b, the equation for the hyperbola is $\frac{x^2}{121} - \frac{y^2}{75} = 1$.

Use the discriminant to identify each conic section.

19. $x^2 + 4y^2 - 2x - 24y + 34 = 0$

SOLUTION:

The equation is written in general form, $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. Find the discriminant.

$$\begin{aligned} B^2 - 4AC &= 0^2 - 4(1)(4) \\ &= -16 \end{aligned}$$

The discriminant is less than 0, so the conic must be either a circle or an ellipse. Because $A \neq C$, the conic is an ellipse.

20. $4x^2 - 25y^2 - 24x - 64 = 0$

SOLUTION:

The equation is written in general form, $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. Find the discriminant.

$$\begin{aligned} B^2 - 4AC &= 0^2 - 4(4)(-25) \\ &= 400 \end{aligned}$$

The discriminant is greater than 0, so the conic is a hyperbola.

21. $2x^2 - y + 5 = 0$

SOLUTION:

The equation is written in general form, $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. Find the discriminant.

$$\begin{aligned} B^2 - 4AC &= 0^2 - 4(2)(0) \\ &= 0 \end{aligned}$$

The discriminant is 0, so the conic is a parabola.

22. $25x^2 + 25y^2 - 100x - 100y + 196 = 0$

SOLUTION:

The equation is written in general form, $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. Find the discriminant.

$$\begin{aligned} B^2 - 4AC &= 0^2 - 4(25)(25) \\ &= -2500 \end{aligned}$$

The discriminant is less than 0, so the conic must be either a circle or an ellipse. Because $A = C$, the conic is a circle.