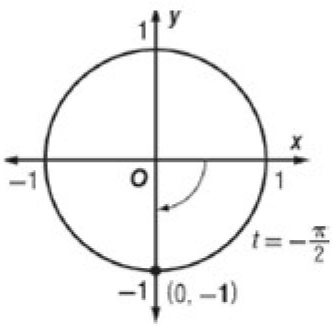
Find the exact value of each expression, if it exists.

53. $\sin^{-1} -1$

SOLUTION:

Find a point on the unit circle on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ with a y-coordinate of –1.



When $t = -\frac{\pi}{2}$, $\sin t = -1$. Therefore, $\sin^{-1} -1 = -\frac{\pi}{2}$.

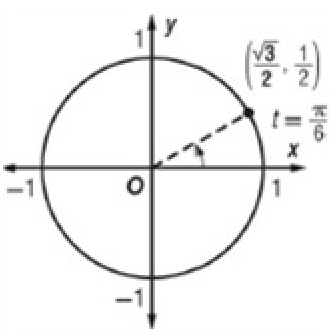
ANSWER:

$-\frac{\pi}{2}$

54. $\cos^{-1} \frac{\sqrt{3}}{2}$

SOLUTION:

Find a point on the unit circle on the interval $[0, \pi]$ with a x-coordinate of $\frac{\sqrt{3}}{2}$.



When $t = \frac{\pi}{6}$, $\cos t = \frac{\sqrt{3}}{2}$. Therefore, $\cos^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{6}$.

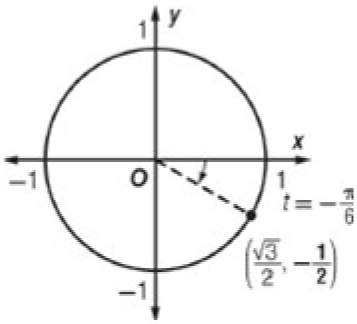
ANSWER:

$\frac{ð}{6}$

55. $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

SOLUTION:

Find a point on the unit circle on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ such that $\frac{y}{x} = -\frac{\sqrt{3}}{3}$



When $t = -\frac{\pi}{6}$, $\tan t = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{3}$. Therefore, $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$.

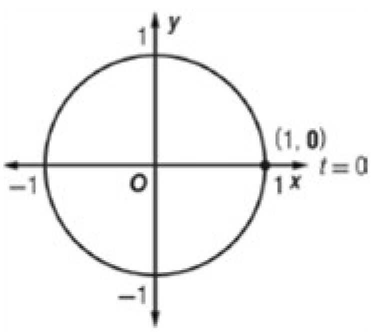
ANSWER:

$-\frac{\pi}{6}$

56. arcsin 0

SOLUTION:

Find a point on the unit circle on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ with a y-coordinate of 0.



When $t = 0$, $\sin t = 0$. Therefore, arcsin $0 = 0$.

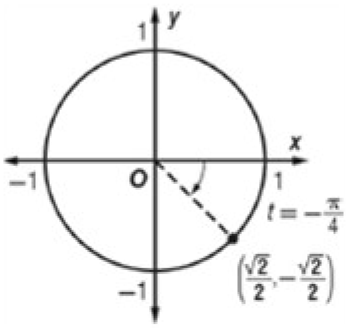
ANSWER:

0

57. arctan −1

SOLUTION:

Find a point on the unit circle on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ such that $\frac{y}{x} = -1$



When $t = -\frac{\pi}{4}$, tan $t = \frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1$. Therefore, arctan $-1 = -\frac{\pi}{4}$.

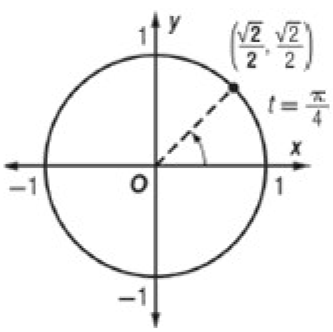
ANSWER:

$-\frac{\pi}{4}$

58. arccos $\frac{\sqrt{2}}{2}$

SOLUTION:

Find a point on the unit circle on the interval $[0, \pi]$ with a x-coordinate of $\frac{\sqrt{2}}{2}$.



When $t = \frac{\pi}{4}$, cos $t = \frac{\sqrt{2}}{2}$. Therefore, arccos $\frac{\sqrt{2}}{2} = \frac{\pi}{4}$.

ANSWER:

$\frac{\delta}{4}$

59. $\sin^{-1}\left[\sin\left(-\frac{\pi}{3}\right)\right]$

SOLUTION:

The inverse property applies, because $-\frac{\pi}{3}$ lies on the interval [–1, 1]. Therefore, $\sin^{-1}\left(\sin -\frac{\pi}{3}\right) = -\frac{\pi}{3}$.

ANSWER:

$-\frac{\pi}{3}$

60. $\cos^{-1}[\cos(-3\pi)]$

SOLUTION:

The inverse property applies, because -3π lies on the interval [–1, 1]. Therefore, $\cos^{-1}(\cos -3\pi) = -3\pi \text{ or } -\pi.$

ANSWER:

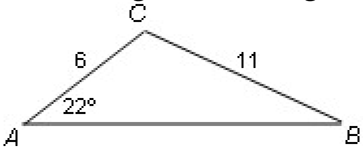
π

Find all solutions for the given triangle, if possible. If no solution exists, write *no solution*. Round side lengths to the nearest tenth and angle measurements to the nearest degree.

61. $a = 11, b = 6, A = 22°$

SOLUTION:

Draw a diagram of a triangle with the given dimensions.



Notice that A is acute and $a > b$ because 11 > 6. Therefore, one solution exists. Apply the Law of Sines to find B.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 22°}{11} = \frac{\sin B}{6}$$
$$6\sin 22° = 11\sin B$$
$$\frac{6\sin 22°}{11} = \sin B$$
$$\sin^{-1}\left(\frac{6\sin 22°}{11}\right) = B$$
$$11.79° \approx B$$

Because two angles are now known, $C \approx 180° - (22° + 12°)$ or about 146°. Apply the Law of Sines to find c.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 22°}{11} = \frac{\sin 146°}{c}$$
$$c\sin 22° = 11\sin 146°$$
$$c = \frac{11\sin 146°}{\sin 22°}$$
$$c \approx 16.4$$

Therefore, the remaining measures of $\triangle ABC$ are
$B \approx 12°$, $C \approx 146°$, and $c \approx 16.4$.

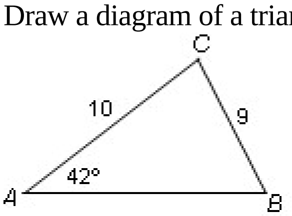
ANSWER:

$B = 12°$, $C = 146°$, $c = 16.4$

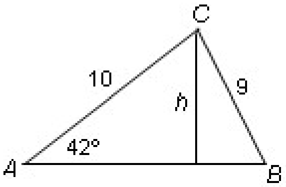
62. $a = 9, b = 10, A = 42°$

SOLUTION:

Draw a diagram of a triangle with the given dimensions.



Notice that A is acute and $a < b$ because 9 < 10. Therefore, two solutions may exist. Find h.



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin 42 = \frac{h}{10}$$
$$10\sin 42 = h$$
$$6.69 \approx h$$

9 > 6.69, so two solutions exist.
Apply the Law of Sines to find *B*.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 42°}{9} = \frac{\sin B}{10}$$
$$10\sin 42° = 9\sin B$$
$$\frac{10\sin 42°}{9} = \sin B$$
$$\sin^{-1}\left(\frac{10\sin 42°}{9}\right) = B$$
$$48° \approx B \text{ or } B = 132°$$

Because two angles are now known, $C \approx 180° - (42° + 48°)$ or about 90°. Apply the Law of Sines to find *c*.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 42°}{9} = \frac{\sin 90°}{c}$$
$$c\sin 42° = 9\sin 90°$$
$$c = \frac{9\sin 90°}{\sin 42°}$$
$$c \approx 13.5$$

When, $B = 132°$ then $C \approx 180° - (42° + 132°)$ or about 6º. Apply the Law of Sines to find *c*.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 42°}{9} = \frac{\sin 6°}{c}$$
$$c\sin 42° = 9\sin 6°$$
$$c = \frac{9\sin 6°}{\sin 42°}$$
$$c \approx 1.4$$

Therefore, the remaining measures of $\triangle ABC$ are
$B \approx 48°$, $C \approx 90°$, and $c \approx 13.5$ or $B = 132°$, $C = 6°$, and $c = 1.4$.

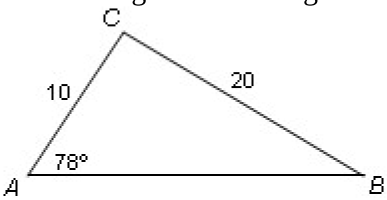
ANSWER:

$B = 48°$, $C = 90°$, $c = 13.5$ and $B = 132°$, $C = 6°$, $c = 1.4$

63. $a = 20$, $b = 10$, $A = 78°$

SOLUTION:

Draw a diagram of a triangle with the given dimensions.



Notice that A is acute and $a > b$ because 20 > 10. Therefore, one solution exists. Apply the Law of Sines to find B.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 78°}{20} = \frac{\sin B}{10}$$
$$10 \sin 78° = 20 \sin B$$
$$\frac{10 \sin 78°}{20} = \sin B$$
$$\sin^{-1}\left(\frac{10 \sin 78°}{20}\right) = B$$
$$29.27° \approx B$$

Because two angles are now known, $C \approx 180° - (78° + 29°)$ or about 73°. Apply the Law of Sines to find c.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 78°}{20} = \frac{\sin 73°}{c}$$
$$c \sin 78° = 20 \sin 73°$$
$$c = \frac{20 \sin 73°}{\sin 78°}$$
$$c \approx 19.5$$

Therefore, the remaining measures of $\triangle ABC$ are $B \approx 29°$, $C \approx 73°$, and $c \approx 19.5$.

ANSWER:

$B = 29°$, $C = 73°$, $c = 19.5$

64. $a = 2$, $b = 9$, $A = 88°$

SOLUTION:

Notice that A is acute and $a < b$ because 2 < 9. Find h.

$$h = b \sin A$$
$$= 9 \sin 88°$$
$$\approx 8.99$$

Because $a < b$ and $a < h$, no triangle can be formed with sides $a = 2$, $b = 9$, and $A = 88°$. Therefore, this problem has no solution.

ANSWER:

no solution

Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

65. $a = 13$, $b = 12$, $c = 8$

SOLUTION:

Use the Law of Cosines to find an angle measure.

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ 13^2 &= 12^2 + 8^2 - 2(12)(8)\cos A° \\ 169 &= 144 + 64 - 192\cos A° \\ -39 &= -192\cos A° \\ \frac{-39}{-192} &= \cos A° \\ \cos^{-1}\left(\frac{-39}{-192}\right) &= A° \\ A° &\approx 78.28° \end{aligned}$$

Use the Law of Sines to find a missing angle measure.

$$\begin{aligned} \frac{\sin A}{a} &= \frac{\sin B}{b} \\ \frac{\sin 78°}{13} &= \frac{\sin B}{12} \\ 12\sin 78° &= 13\sin B \\ \frac{12\sin 78°}{13} &= \sin B \\ \sin^{-1}\left(\frac{12\sin 78°}{13}\right) &= B \\ 64.54° &\approx B \end{aligned}$$

Find the measure of the remaining angle.

$$\begin{aligned} C &\approx 180° - (78° + 65°) \\ &\approx 37° \end{aligned}$$

Therefore, $A \approx 78°$, $B \approx 65°$, and $C \approx 37°$.

ANSWER:

$A = 78°$, $B = 65°$, $C = 37°$

66. $a = 4$, $b = 5$, $C = 96°$

SOLUTION:

Use the Law of Cosines to find the missing side measure.

$$c^2 = a^2 + b^2 - 2ab\cos A$$
$$c^2 = 4^2 + 5^2 - 2(4)(5)\cos 96°$$
$$c^2 = 16 + 25 - 40\cos 96°$$
$$c^2 = 45.18$$
$$c \approx 6.72$$

Use the Law of Sines to find a missing angle measure.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin A°}{4} = \frac{\sin 96°}{6.7}$$
$$6.7\sin A° = 4\sin 96°$$
$$\sin A° = \frac{4\sin 96°}{6.7}$$
$$A = \sin^{-1}\left(\frac{4\sin 96°}{6.7}\right)$$
$$A \approx 36.42°$$

Find the measure of the remaining angle.

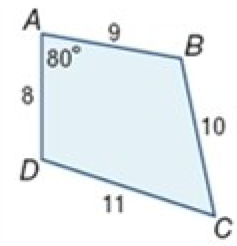
$$C \approx 180° - (96° + 36°)$$
$$\approx 48°$$

Therefore, $c = 6.7, A = 36°$, and $B = 48°$.

ANSWER:

$c = 6.7, A = 36°, B = 48°$

77. **GEOMETRY** Consider quadrilateral *ABCD*.



a. Find *C*.
b. Find the area of *ABCD*.

SOLUTION:

a. Use SAS to find the length of *BD*.

$$BD^2 = 8^2 + 9^2 - 2(8)(9)\cos 80$$
$$BD^2 = 64 + 81 - 144\cos 80$$
$$BD = \sqrt{145 - 144\cos 80}$$
$$BD \approx 10.95$$

Use SSS to find *C*.

$$10.95^2 = 10^2 + 11^2 - 2(10)(11)\cos A$$
$$120 \approx 100 + 121 - 220\cos A$$
$$-101 \approx -220\cos A$$
$$\cos^{-1}\left(\frac{101}{220}\right) \approx A$$
$$63° \approx A$$

b. Use SAS to find the area of each triangle formed by *BD*.

$$\text{Area} = \frac{1}{2}(8)(9)\sin 80$$
$$= 36\sin 80$$
$$\approx 35.45$$
$$\text{Area} = \frac{1}{2}(10)(11)\sin 63$$
$$= 55\sin 63$$
$$\approx 49$$
$$35.45 + 49 \approx 84$$

ANSWER:

a. 63°
b. 84 units^2