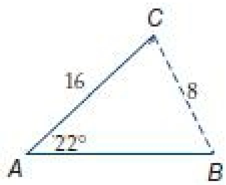
Find all solutions for the given triangle, if possible. If no solution exists, write *no solution*. Round side lengths to the nearest tenth and angle measurements to the nearest degree.

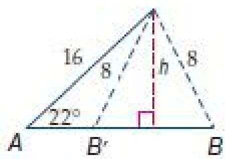
19. $a = 8$, $b = 16$, $A = 22°$

SOLUTION:

Draw a diagram of a triangle with the given dimensions.



Notice that A is acute and $a < b$ because 8< 16. Therefore, two solutions may exist. Find h.



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin 22 = \frac{h}{16}$$
$$16\sin 22 = h$$
$$6 \approx h$$

8 > 6, so two solutions exist.

Apply the Law of Sines to find B.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 22°}{8} = \frac{\sin B}{16}$$
$$16\sin 22° = 8\sin B$$
$$\frac{16\sin 22°}{8} = \sin B$$
$$\sin^{-1}\left(\frac{16\sin 22°}{8}\right) = B$$
$$49° \approx B \text{ or } B \approx 131°$$

Because two angles are now known, $C \approx 180° - (22° + 49°)$ or about 109°. Apply the Law of Sines to find c.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 22}{8} = \frac{\sin 109}{c}$$
$$c\sin 22 = 8\sin 109$$
$$c = \frac{8\sin 109}{\sin 22}$$
$$c \approx 20.1$$

When $B \approx 131°$, then $C \approx 180° - (22° + 131°)$ or about 27°. Apply the Law of Sines to find c.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 22}{8} = \frac{\sin 27}{c}$$
$$c \sin 22 = 8 \sin 27$$
$$c = \frac{8 \sin 27}{\sin 22}$$
$$c \approx 9.7$$

Therefore, the remaining measures of $\triangle ABC$ are $B \approx 49°$, $C \approx 109°$, $c \approx 20.1$ and $B \approx 131°$, $C \approx 27°$, $c \approx 9.7$.

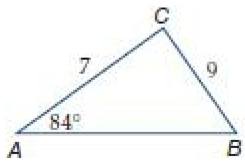
ANSWER:

$B = 49°$, $C = 109°$, $c = 20.1$ and $B = 131°$, $C = 27°$, $c = 9.7$

20. $a = 9$, $b = 7$, $A = 84°$

SOLUTION:

Draw a diagram of a triangle with the given dimensions.



Notice that A is acute and $a > b$ because $9 > 7$. Therefore, one solution exists. Apply the Law of Sines to find B.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 84}{9} = \frac{\sin B}{7}$$
$$7\sin 84 = 9\sin B$$
$$\frac{7\sin 84}{9} = \sin B$$
$$\sin^{-1}\left(\frac{7\sin 84}{9}\right) = B$$
$$50.96° \approx B$$

Because two angles are now known, $C \approx 180° - (84° + 51°)$ or about $45°$. Apply the Law of Sines to find c.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 84}{9} = \frac{\sin 45}{c}$$
$$c\sin 84 = 9\sin 45$$
$$c = \frac{9\sin 45}{\sin 84}$$
$$c \approx 6.4$$

Therefore, the remaining measures of $\triangle ABC$ are
$B \approx 51°$, $C \approx 45°$, and $c \approx 6.4$.

ANSWER:

$B = 51°$, $C = 45°$, $c = 6.4$

21. $a = 3, b = 5, c = 7$

SOLUTION:

Use the Law of Cosines to find an angle measure.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$3^2 = 5^2 + 7^2 - 2(5)(7)\cos A°$$
$$9 = 25 + 49 - 70\cos A°$$
$$-65 = -70\cos A°$$
$$\frac{-65}{-70} = \cos A°$$
$$\cos^{-1}\left(\frac{-65}{-70}\right) = A°$$
$$A° \approx 21.79°$$

Use the Law of Sines to find a missing angle measure.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 21.79}{3} = \frac{\sin B}{5}$$
$$5\sin 21.79 = 3\sin B$$
$$\frac{5\sin 21.79}{3} = \sin B$$
$$\sin^{-1}\left(\frac{5\sin 21.79}{3}\right) = B$$
$$38.22° \approx B$$

Find the measure of the remaining angle.

$$C \approx 180° - (22° + 38°)$$
$$\approx 120°$$

Therefore, $A \approx 22°$, $B \approx 38°$, and $C \approx 120°$.

ANSWER:

$A = 22°$, $B = 38°$, $C = 120°$

22. $a = 8, b = 10, C = 46°$

SOLUTION:

Use the Law of Cosines to find the missing side measure.

$$c^2 = a^2 + b^2 - 2ab\cos A$$
$$c^2 = 8^2 + 10^2 - 2(8)(10)\cos 46°$$
$$c^2 = 64 + 100 - 160\cos 46°$$
$$c^2 = 52.85$$
$$c \approx 7.27$$

Use the Law of Sines to find a missing angle measure.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin A°}{8} = \frac{\sin 46°}{7.3}$$
$$7.3\sin A° = 8\sin 46°$$
$$\sin A° = \frac{8\sin 46°}{7.3}$$
$$A = \sin^{-1}\left(\frac{8\sin 46°}{7.3}\right)$$
$$A \approx 52.03°$$

Find the measure of the remaining angle.

$$C \approx 180° - (46° + 52°)$$
$$\approx 82°$$

Therefore, $c \approx 7.3$, $A \approx 52°$, and $B \approx 82°$.

ANSWER:

$c = 7.3, A = 52°, B = 82°$

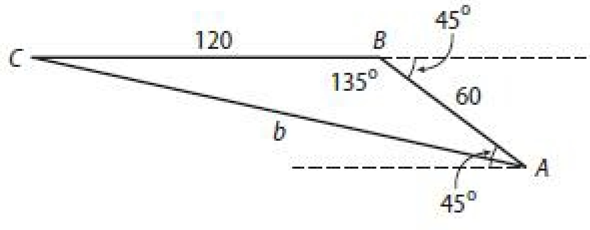
25. **NAVIGATION** A boat leaves a dock and travels 45º north of west averaging 30 knots for 2 hours. The boat then travels directly west averaging 40 knots for 3 hours.

a. How many nautical miles is the boat from the dock after 5 hours?
b. How many degrees south of east is the dock from the boat's present position?

SOLUTION:

a. During the first leg of the trip the boat traveled 30 knots for 2 hours, and therefore traveled a distance of 30 · 2 or 60 nautical miles. During the second leg of the trip, the boat traveled 40 knots for 3 hours, so the distance the boat traveled is 40 · 3 or 120 nautical miles.

Draw a diagram to model the situation. Let x represent the distance the boat has traveled from the dock after 5 hours.



Use the Law of Cosines to find *b*.

$$b^2 = 120^2 + 60^2 - 2(120)(60)\cos 135°$$
$$b = \sqrt{14400 + 3600 - 14400\cos 135°}$$
$$b \approx 167.9$$

Therefore, the boat is 167.9 nautical miles from the dock after 5 hours.

b. Use the Law of Sines to find *C*.

$$\frac{\sin 135°}{167.9} = \frac{\sin C}{60}$$
$$60 \sin 135° = 167.9 \sin C$$
$$\frac{60 \sin 135°}{167.9} = \sin C$$
$$\sin^{-1}\left(\frac{60 \sin 135°}{167.9}\right) = C$$
$$14.6 \approx C$$

Therefore, the dock is about 15° south of east from the boat's current position.

ANSWER:

a. about 167.9 nautical mi

b. about 15° south of east