Study Guide and Review - Chapter 1

Determine whether each relation represents y as a function of x.

11. $3x - 2y = 18$

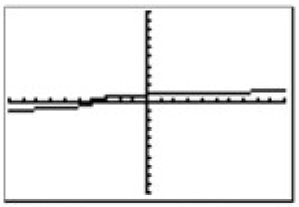
SOLUTION:

This equation represents y as a function of x, because for every x-value there is exactly one corresponding y-value. The only linear equations that are not functions are of the form $x = c$.

12. $y^3 - x = 4$

SOLUTION:

This equation represents y as a function of x, because for every x-value there is exactly one corresponding y-value. A sketch of the graph shows that the function passes the Vertical Line Test.



[−10, 10] scl: 1 by [−10, 10] scl: 1

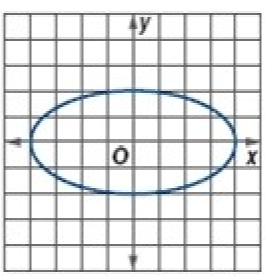
13.

x	y
5	7
7	9
9	11
11	13

SOLUTION:

This equation represents y as a function of x, because for every x-value there is exactly one corresponding y-value.

14.



SOLUTION:

This graph does not represent a function because it fails the Vertical Line Test.

Let $f(x) = x^2 - 3x + 4$. Find each function value.

15. $f(5)$

SOLUTION:

$$\begin{aligned} f(5) &= (5)^2 - 3(5) + 4 \\ &= 25 - 15 + 4 \\ &= 14 \end{aligned}$$

16. $f(-3x)$

SOLUTION:

$$\begin{aligned} f(5) &= (-3x)^2 - 3(-3x) + 4 \\ &= 9x^2 + 9x + 4 \end{aligned}$$

State the domain of each function.

17. $f(x) = 5x^2 - 17x + 1$

SOLUTION:

There is no value of x that will make the function undefined, so D = $\{x \mid x \in \mathrm{R}\}$.

18. $g(x) = \sqrt{6x - 3}$

SOLUTION:

The function is undefined when $6x - 3 < 0$.

$$\begin{aligned} 6x - 3 &< 0 \\ 6x &< 3 \\ x &< \frac{1}{2} \\ x &< 0.5 \end{aligned}$$

D = $\{x \mid x \geq 0.5, x \in \mathrm{R}\}$

19. $h(a) = \frac{5}{a+5}$

SOLUTION:

The function is undefined when $a + 5 = 0$.

D = $\{a \mid a \neq -5, a \in \mathrm{R}\}$

20. $v(x) = \frac{x}{x2 - 4}$

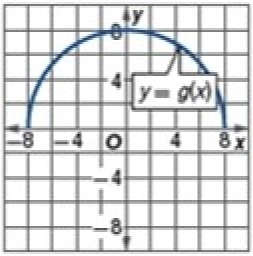
SOLUTION:

The function is undefined when $x^2 - 4 = 0$.

$$\begin{aligned} x^2 - 4 &= 0 \\ x^2 &= 4 \\ x &= \pm 2 \end{aligned}$$

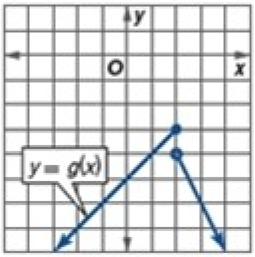
D = $\{x \mid x \neq \pm 2, x \in \mathrm{R}\}$

Use the graph of *g* to find the domain and range of each function.

21. 

SOLUTION:

The *x*-values range from −8 to 8 and the *y*-values range from 0 to 8. D = [−8, 8], R = [0, 8]

22. 

SOLUTION:

The arrows indicate that the *x*-values extend to negative infinity and positive infinity. The *y*-values extend to negative infinity and reach a maximum of −3.
D = {*x* | *x* ∈ R}, R = (−∞ , −3)

Find the *y*-intercept(s) and zeros for each function.

23. $f(x) = 4x - 9$

SOLUTION:

$$\begin{aligned} f(0) &= 4(0) - 9 \\ &= 0 - 9 \\ &= -9 \end{aligned}$$

$$\begin{aligned} 4x - 9 &= 0 \\ 4x &= 9 \\ x &= \frac{9}{4} \end{aligned}$$

24. $f(x) = x^2 - 6x - 27$

SOLUTION:

$$\begin{aligned} f(0) &= (0)^2 - 6(0) - 27 \\ &= 0 - 0 - 27 \\ &= -27 \end{aligned}$$

$$\begin{aligned} x^2 - 6x - 27 &= 0 \\ (x - 9)(x + 3) &= 0 \\ x &= 9 \text{ or } -3 \end{aligned}$$

25. $f(x) = x^3 - 16x$

SOLUTION:

$$f(0) = (0)^3 - 16(0) \\ = 0 - 0 \\ = 0$$

$$x^3 - 16x = 0 \\ x(x^2 - 16) = 0 \\ x(x+4)(x-4) = 0 \\ x = 0, 4, \text{ or } -4$$

26. $f(x) = \sqrt{x+2} - 1$

SOLUTION:

$$f(0) = \sqrt{0+2} - 1 \\ = \sqrt{2} - 1$$

$$\sqrt{x+2} - 1 = 0 \\ \sqrt{x+2} = 1 \\ x + 2 = 1 \\ x = -1$$

Determine whether each function is continuous at the given x-value(s). Justify using the continuity test. If discontinuous, identify the type of discontinuity as *infinite*, *jump*, or *removable*.

27. $f(x) = x^2 - 3x$; $x = 4$

SOLUTION:

Find $f(4)$.

$$f(x) = x^2 - 3x$$

$$\begin{aligned} f(4) &= (4)^2 - 3(4) \\ &= 16 - 12 \\ &= 4 \end{aligned}$$

The function is defined at $x = 4$.

Find $\lim_{x \to 4} f(x)$. Construct a table that shows values of $f(x)$ for x-values approaching 4 from the left and from the right.

x	$f(x)$
3.9	3.51
3.99	3.9501
3.999	3.995
4	4
4.001	4.005
4.01	4.0501
4.1	4.51

$\lim_{x \to 4} f(x) = 4$.

Because $\lim_{x \to 4} f(x) = f(4)$, $f(x)$ is continuous as $x = 4$.

28. $f(x) = \sqrt{2x-4}$; $x = 10$

SOLUTION:

Find $f(10)$.

$$f(x) = \sqrt{2x-4}$$
$$f(1) = \sqrt{2(10)-4}$$
$$= \sqrt{20-4}$$
$$= \sqrt{16}$$
$$= 4$$

The function is defined at $x = 10$.

Find $\lim_{x \to 10} f(x)$. Construct a table that shows values of $f(x)$ for x-values approaching 10 from the left and from the right.

x	$f(x)$
9.9	3.975
9.99	3.9975
9.999	3.9997
10	4
10.001	4.0002
10.01	4.0025
10.1	4.025

$\lim_{x \to 10} f(x) = 4$.

Because $\lim_{x \to 10} f(x) = f(10)$, $f(x)$ is continuous as $x = 10$.

29. $f(x) = \frac{x}{x+7}$; $x = 0$ and $x = 7$

SOLUTION:

Find $f(0)$.

$$f(x) = \frac{x}{x+7}$$
$$f(0) = \frac{0}{0+7}$$
$$= \frac{0}{7}$$
$$= 0$$

The function is defined at $x = 0$.

Find $\lim_{x \to 0} f(x)$. Construct a table that shows values of $f(x)$ for x-values approaching 0 from the left and from the right.

x	$f(x)$

−0.1	−0.0145
−0.01	−0.0014
−0.001	−0.0001
0	0
0.001	0.0001
0.01	0.0014
0.1	0.01408

$\lim_{x \to 0} f(x) = 0$.

Because $\lim_{x \to 0} f(x) = f(0)$, $f(x)$ is continuous as $x = 0$.

Find $f(7)$.

$$f(x) = \frac{x}{x+7}$$

$$f(7) = \frac{7}{7+7}$$

$$= \frac{7}{14}$$

$$= 0.5$$

The function is defined at $x = 7$.

Find $\lim_{x \to 7} f(x)$. Construct a table that shows values of $f(x)$ for x-values approaching 7 from the left and from the right.

x	$f(x)$
6.9	0.4964
6.99	0.49964
6.999	0.49996
7	0.5
7.001	0.50004
7.01	0.50036
7.1	0.50355

$\lim_{x \to 7} f(x) = 0.5$.

Because $\lim_{x \to 7} f(x) = f(7)$, $f(x)$ is continuous as $x = 7$.

30. $f(x) = \frac{x}{x2 - 4}$; $x = 2$ and $x = 4$

SOLUTION:

Find $f(2)$.

$$f(x) = \frac{x}{x^2 - 4}$$
$$f(2) = \frac{2}{2^2 - 4}$$
$$= \frac{2}{0}$$

The function is undefined at x = 2.
Find $\lim_{x \to 2} f(x)$. Construct a table that shows values of $f(x)$ for x-values approaching 2 from the left and from the right.

x	$f(x)$
1.9	−4.872
1.99	−49.87
1.999	−499.9
2	undefined
2.001	500.12
2.01	50.125
2.1	5.122

Because $f(2)$ is undefined and $f(x)$ approaches $-\infty$ as x approaches 2 from the left and ∞ as x approaches 2 from the right, $f(x)$ is discontinuous at x = 2 and has an infinite discontinuity at x = 2.

Find $f(4)$.

$$f(x) = \frac{x}{x^2 - 4}$$
$$f(4) = \frac{4}{4^2 - 4}$$
$$= \frac{4}{16 - 4}$$
$$= \frac{4}{12}$$
$$= \frac{1}{3}$$

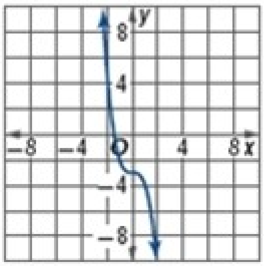
The function is defined at x = 4.
Find $\lim_{x \to 4} f(x)$. Construct a table that shows values of $f(x)$ for x-values approaching 4 from the left and from the right.

x	$f(x)$
3.9	0.3479
3.99	0.33473
3.999	0.33347
4	0.33333
4.001	0.33319
4.01	0.33195
4.1	0.32006

$\lim_{x \to 4} f(x) = \frac{1}{3}$.

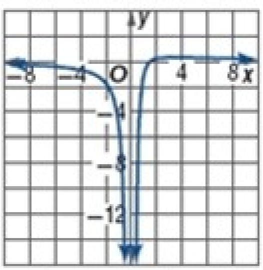
Because $\lim_{x \to 4} f(x) = f(4)$, $f(x)$ is continuous as $x = 4$.

Use the graph of each function to describe its end behavior.

32. 

SOLUTION:

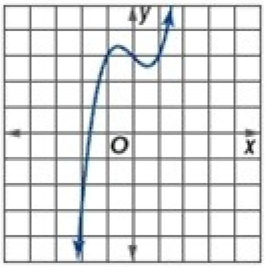
Analyze the graph from left to right. The left side of the graph indicates where x approaches negative infinity while the right side indicates where x approaches positive infinity. From the graph, it appears that as $x \to \infty$, $f(x) \to -\infty$; as $x \to -\infty$, $f(x) \to \infty$.

33. 

SOLUTION:

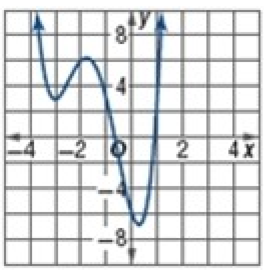
Analyze the graph from left to right. The left side of the graph indicates where x approaches negative infinity while the right side indicates where x approaches positive infinity. From the graph, it appears that as $x \to \infty$, $f(x) \to 0$; as $x \to -\infty$, $f(x) \to 0$.

Use the graph of each function to estimate intervals to the nearest 0.5 unit on which the function is increasing, decreasing, or constant. Then estimate to the nearest 0.5 unit, and classify the extrema for the graph of each function.

34. 

SOLUTION:

Analyze the graph from left to right. As the graph changes from increasing to decreasing, there is a maximum. As the graph changes from decreasing to increasing, there is a minimum. When the extremum is the farthest point on the graph, then it is an absolute extremum. f is increasing on $(-\infty, -0.5)$, decreasing on $(-0.5, 0.5)$, and increasing on $(0.5, \infty)$; relative maximum at $(-0.5, 3.5)$ and relative minimum at $(0.5, 2.5)$.

35. 

SOLUTION:

Analyze the graph from left to right. As the graph changes from increasing to decreasing, there is a maximum. As the graph changes from decreasing to increasing, there is a minimum. When the extremum is the farthest point on the graph, then it is an absolute extremum. f is decreasing on $(-\infty, -3)$, increasing on $(-3, -1.5)$, decreasing on $(-1.5, 0.5)$, and increasing on $(0.5, \infty)$; relative minimum at $(-3, 3)$, relative maximum at $(-1.5, 6)$ and relative minimum at $(0.5, -7)$.

Find the average rate of change of each function on the given interval.

36. $f(x) = -x^3 + 3x + 1$; $[0, 2]$

SOLUTION:

$$\begin{aligned}\frac{f(b)-f(a)}{b-a} &= \frac{-(2)^3+3(2)+1-[-(0)^3+3(0)+1]}{2-0}\\ &= \frac{-8+6+1-1}{2}\\ &= \frac{-2}{2}\\ &= -1\end{aligned}$$

37. $f(x) = x^2 + 2x + 5$; [−5, 3]

SOLUTION:

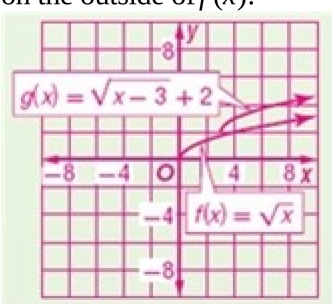
$$\begin{aligned}\frac{f(b)-f(a)}{b-a} &= \frac{(3)^2+2(3)+5-[(-5)^2+2(-5)+5]}{3-(-5)}\\ &= \frac{9+6+5-[25-10+5]}{8}\\ &= \frac{20-[20]}{8}\\ &= 0\end{aligned}$$

Identify the parent function $f(x)$ of $g(x)$, and describe how the graphs of $g(x)$ and $f(x)$ are related. Then graph $f(x)$ and $g(x)$ on the same axes.

38. $g(x) = \sqrt{x-3} + 2$

SOLUTION:

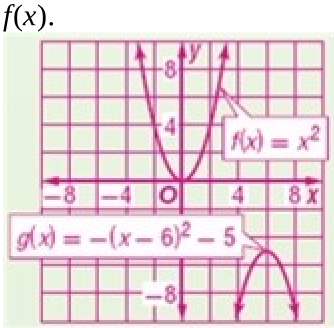
$g(x) = f(x - 3) + 2$, so $g(x)$ is the graph of $f(x) = \sqrt{x}$ translated 3 units to the right and 2 units up. The translation right is represented by the subtraction of 3 inside the function. The translation up is represented by the addition of 2 on the outside of $f(x)$.



39. $g(x) = -(x - 6)^2 - 5$

SOLUTION:

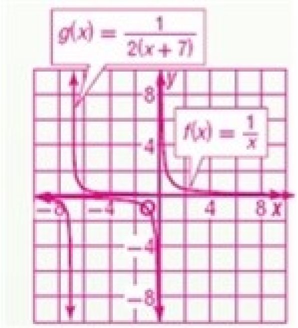
$g(x) = -f(x - 6) - 5$, so $g(x)$ is the graph of $f(x) = x^2$ reflected in the x-axis and translated 6 units to the right and 5 units down. The translation right is represented by the subtraction of 6 inside the function. The reflection is represented by the negative coefficient of $f(x)$. The translation down is represented by the subtraction of 5 outside of $f(x)$.



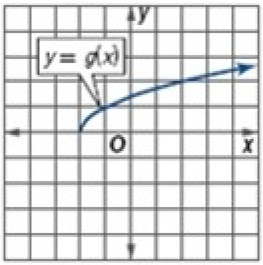
40. $g(x) = \frac{1}{2(x+7)}$

SOLUTION:

$g(x) = \frac{1}{2}f(x + 7)$, so $g(x)$ is the graph of $f(x) = \frac{1}{x}$ translated 7 units to the left and is compressed vertically by a factor of $\frac{1}{2}$. The translation is represented by the addition of 7 inside the function. The compression is represented by the coefficient of $f(x)$.

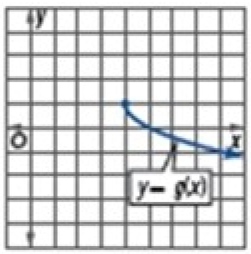


Describe how the graphs of $f(x) = \sqrt{x}$ and $g(x)$ are related. Then write an equation for $g(x)$.

42. 

SOLUTION:

The graph is translated 2 units left. This is done by adding 2 inside the function. $g(x) = f(x + 2) = \sqrt{x+2}$.

43. 

SOLUTION:

The graph is reflected in the x-axis and translated 4 units right and 1 unit up. This is done by multiplying $f(x)$ by −1, subtracting 4 inside $f(x)$ and adding 1 outside $f(x)$. $g(x) = -f(x - 4) + 1 = -\sqrt{x-4} + 1$.

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. State the domain of each new function.

44. $f(x) = x + 3$
$g(x) = 2x^2 + 4x - 6$

SOLUTION:

$$\begin{aligned}(f+g)(x) &= x+3+2x^2+4x-6\\ &= 2x^2+5x-3\\ (f-g)(x) &= x+3+-(2x^2+4x-6)\\ &= -2x^2-3x+9\\ (f\cdot g)(x) &= (x+3)(2x^2+4x-6)\\ &= 2x^3+4x^2-6x+6x^2+12x-18\\ &= 2x^3+10x^2+6x-18\\ \left(\frac{f}{g}\right)(x) &= \frac{x+3}{2x^2+4x-6}\\ &= \frac{x+3}{(2x-2)(x+3)}\\ &= \frac{1}{2x-2}\end{aligned}$$

D = $(-\infty, \infty)$ for all of the functions except $\left(\frac{f}{g}\right)(x)$, for which $D = (-\infty, -3) \cup (-3, 1) \cup (1, \infty)$. Even though there appears to be no restriction of −3 in the simplified function, there is in the original.

For each pair of functions, find $[f \circ g](x)$, $[g \circ f](x)$, and $[f \circ g](2)$.

48. $f(x) = 4x - 11$; $g(x) = 2x^2 - 8$

SOLUTION:

$$\begin{aligned}[f\circ g](x) &= 4(2x^2-8)-11\\ &= 8x^2-32-11\\ &= 8x^2-43\\ [g\circ f](x) &= 2(4x-11)^2-8\\ &= 2(16x^2-88x+121)-8\\ &= 32x^2-176x+242-8\\ &= 32x^2-176x+234\\ [f\circ g](6) &= 8(2)^2-43\\ &= 32-43\\ &= -11\end{aligned}$$

Find $f \circ g$.

52. $f(x) = \sqrt{x-2}$
$g(x) = 6x - 7$

SOLUTION:

The domain of $f(x) = x > 2$. In order for the range of $g(x)$ to correspond with this domain, $g(x)$ must be greater than or equal to 2.

$$6x - 7 > 2$$
$$6x > 9$$
$$x > \frac{3}{2}$$
$$[f \circ g](x) = \sqrt{(6x-7)-2}$$
$$= \sqrt{6x-9}$$

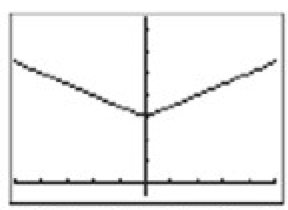
There are no more restrictions.

$[f \circ g](x) = \sqrt{6x-9}$ for $x \geq \frac{3}{2}$

Graph each function using a graphing calculator, and apply the horizontal line test to determine whether its inverse function exists. Write *yes* or *no*.

53. $f(x) = |x| + 6$

SOLUTION:

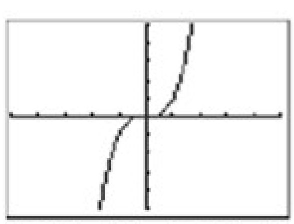


[−5, 5] scl: 1 by [−1, 15] scl: 2

This graph fails the Horizontal Line Test.

54. $f(x) = x^3$

SOLUTION:

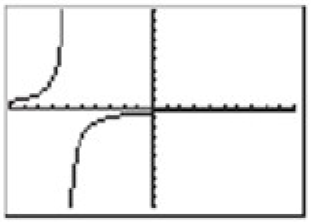


[−5, 5] scl: 1 by [−5, 5] scl: 1

This graph passes the Horizontal Line Test.

55. $f(x) = -\frac{3}{x+6}$

SOLUTION:

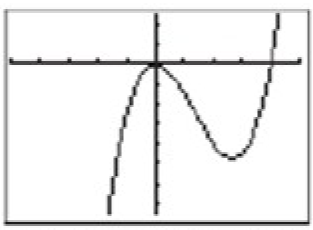


[–10, 10] scl: 1 by [–10, 10] scl: 1

This graph passes the Horizontal Line Test.

56. $f(x) = x^3 - 4x^2$

SOLUTION:



[–5, 5] scl: 1 by [–15, 5] scl: 2

This graph fails the Horizontal Line Test.

Find the inverse function and state any restrictions on the domain.

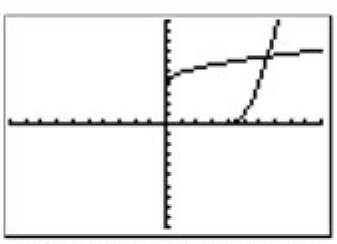
57. $y = (x - 4)^2$

SOLUTION:

Interchange the variables and solve for y.

$$x = (y-4)^2$$
$$\pm\sqrt{x} = y - 4$$
$$\pm\sqrt{x} + 4 = y$$

Graph the functions.



[–10, 10] scl: 1 by [–10, 10] scl: 1

From the graph, the domain and range of f correspond with the range and domain of f^{-1} when the domain of f is restricted to $\{x | x \geq 4\}$.

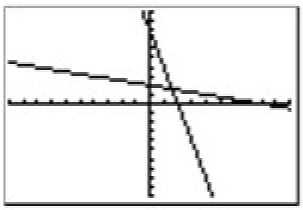
58. $y = -4x + 8$

SOLUTION:

Interchange the variables and solve for y.

$$x = -4y + 8$$
$$x - 8 = -4y$$
$$\frac{x-8}{-4} = y$$
$$-\frac{1}{4}x + 2 = y$$

Graph the functions.



[−10, 10] scl: 1 by [−10, 10] scl: 1

From the graph, the domain and range of f correspond with the range and domain of f^{-1} for all real numbers.

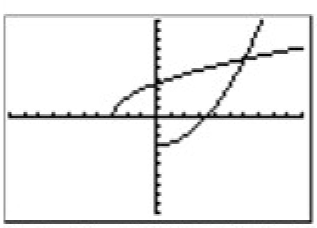
59. $y = 2\sqrt{x+3}$

SOLUTION:

Interchange the variables and solve for y.

$$x = 2\sqrt{y+3}$$
$$\frac{x}{2} = \sqrt{y+3}$$
$$\frac{x^2}{4} = y + 3$$
$$\frac{1}{4}x^2 - 3 = y$$

Graph the functions.



[−10, 10] scl: 1 by [−10, 10] scl: 1

From the graph, the domain and range of f correspond with the range and domain of f^{-1} when the domain of f is restricted to $\{x \mid x \geq -3\}$.

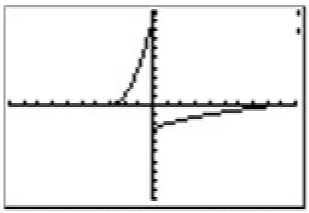
60. $y = \sqrt{x} - 3$

SOLUTION:

Interchange the variables and solve for y.

$$x = \sqrt{y} - 3$$
$$x + 3 = \sqrt{y}$$
$$(x+3)^2 = y$$

Graph the functions.



[−10, 10] scl: 1 by [−10, 10] scl: 1

From the graph, the domain and range of f correspond with the range and domain of f^{-1} when the domain of f is restricted to $\{x|\ x \geq 0\}$.

61. **CELL PHONES** Basic Mobile offers a cell phone plan that charges $39.99 per month. Included in the plan are 500 daytime minutes that can be used Monday through Friday between 7 A.M. and 7 P.M. Users are charged $0.20 per minute for every daytime minute over 500 used.

a. Write a function $p(x)$ for the cost of a month of service during which you use x daytime minutes.

b. How much will you be charged if you use 450 daytime minutes? 550 daytime minutes?

c. Graph $p(x)$.

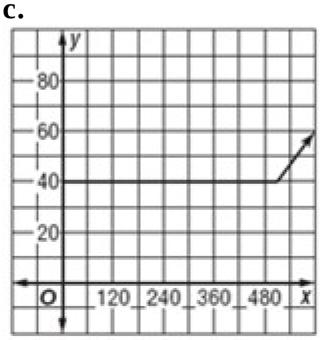
SOLUTION:

a. There is a base charge of $39.99 which you will pay regardless of the number of minutes you use. You also receive 500 *free* minutes, so if you use 500 or less minutes, then you will only be charged the base fee. Once you go over 500 minutes, then you will be charged the base fee plus 0.20x$ where x is the number of minutes over 500. This can be written using a piecewise function.

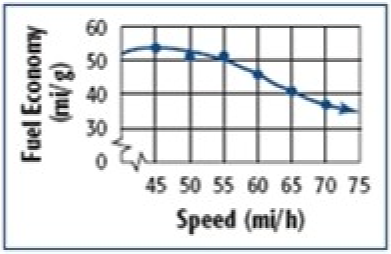
$$f(x) = \begin{cases} 39.99 & \text{if } 0 \leq x \leq 500 \\ 39.99 + 0.2(x - 500) & \text{if } x > 500 \end{cases}$$

b. For 450 minutes, you will only be charged the base fee. For 550 minutes, you will be charge the base fee plus $0.20(50). $39.99; $49.99

c.



62. **AUTOMOBILES** The fuel economy for a hybrid car at various highway speeds is shown.



a. Approximately what is the fuel economy for the car when traveling 50 miles per hour?
b. At approximately what speed will the car's fuel economy be less than 40 miles per gallon?

SOLUTION:

a. Analyze the graph. The speed is shown along the *x*-axis. Scroll to where the speed is 50 mph and then look up to where the graph is at that point. It appears to be near 51 mi/g.

Sample answer: approximately 51 mpg

b. Analyze the graph. The fuel economy is shown along the *y*-axis. Scroll to where the fuel economy is 40 mi/g and then look to where the graph is at that point. It appears to be near 67 mph.
Sample answer: approximately 67 mph or faster

63. SALARIES After working for a company for five years, Ms. Washer was given a promotion. She is now earning $1500 per month more than her previous salary. Will a function modeling her monthly income be a continuous function? Explain.

SOLUTION:

Before her promotion, the graph of her salary is constant and will be represented by a horizontal line. After the promotion, the graph will jump up and then be represented by a new horizontal line. There will be a jump discontinuity at this point.

64. **BASEBALL** The table shows the number of home runs by a baseball player in each of the first 5 years he played professionally.

Year	2004	2005	2006	2007	2008
Number of Home Runs	5	36	23	42	42

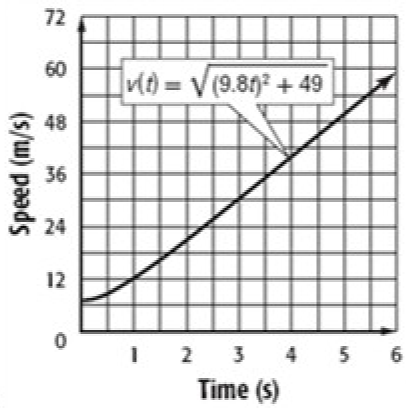
a. Explain why 2006 represents a relative minimum.
b. Suppose the average rate of change of home runs between 2008 and 2011 is 5 home runs per year. How many home runs were there in 2011?
c. Suppose the average rate of change of home runs between 2007 and 2012 is negative. Compare the number of home runs in 2007 and 2012.

SOLUTION:

a. Sample answer: The number of home runs decreased, then increased. This tells us that there is a minimum. Also, the value of 23 is not the smallest value on the graph. Therefore, the minimum is local and not absolute.
b. $42 \cdot 3(5) = 57$
c. When the average rate of change is negative from point *a* to point *b*, then the graph has decreased from *a* to *b*. There were fewer home runs in 2012 than in 2007.

65. **PHYSICS** A stone is thrown horizontally from the top of a cliff. The velocity of the stone measured in meters per second after t seconds can be modeled by $v(t) = –9.8t + 7$. The speed of the stone is the absolute value of its velocity. Draw a graph of the stone's speed during the first 6 seconds.

SOLUTION:



Graph $y = \sqrt{(9.8x)2+49}$