Practice Test - Chapter 2

Solve each equation.

7. $x^4 - 5x^3 - 14x^2 = 0$

SOLUTION:

The equation can be written as $x^2(x^2 - 5x - 14) = 0$ or $x^2(x - 7)(x + 2) = 0$. The solutions are 0, 7, and −2.

ANSWER:

0, 7, –2

Describe the end behavior of the graph of each polynomial function using limits. Explain your reasoning using the leading term test.

9. $f(x) = 5x^4 - 3x^3 - 7x^2 + 11x - 8$

SOLUTION:

$\lim_{x \to \infty} f(x) = \infty$; $\lim_{x \to -\infty} f(x) = \infty$; The degree is even and the leading coefficient is positive.

ANSWER:

$\lim_{x \to \infty} f(x) = \infty$; $\lim_{x \to -\infty} f(x) = \infty$; The degree is even and the leading coefficient is positive.

10. $f(x) = -3x^5 - 8x^4 + 7x^2 + 5$

SOLUTION:

$\lim_{x \to \infty} f(x) = -\infty$; $\lim_{x \to -\infty} f(x) = \infty$; The degree is odd and the leading coefficient is negative.

ANSWER:

$\lim_{x \to \infty} f(x) = -\infty$; $\lim_{x \to -\infty} f(x) = \infty$; The degree is odd and the leading coefficient is negative.

State the number of possible real zeros and turning points of each function. Then find all of the real zeros by factoring.

11. $f(x) = 4x^3 + 8x^2 - 60x$

SOLUTION:

The degree of $f(x)$ is 3, so it will have at most three real zeros and two turning points.

$$0 = 4x^3 + 8x^2 - 60x$$

$$0 = 4x(x^2 + 2x - 15)$$

$$0 = x^3(x + 5)(x - 3)$$

So, the zeros are −5, −0, and 3.

ANSWER:

3 real zeros and 2 turning points; –5, 0, and 3

12. $f(x) = x^5 - 16x$

SOLUTION:

The degree of $f(x)$ is 5, so it will have at most five real zeros and four turning points.

$$0 = x^5 - 16x$$

$$0 = x(x^4 - 16)$$

$$0 = x(x^2 + 4)(x^2 - 4)$$

$$0 = x(x - 2)(x + 2)(x^2 + 4)$$

$x^2 + 4$ yields no real zeros. So, the zeros are −2, 0, and 2.

ANSWER:

5 real zeros and 4 turning points; –2, 0, and 2

13. **MULTIPLE CHOICE** Which function has 3 turning points?

A $f(x) = x^4 - 4$

B $f(x) = x^4 - 11x^3$

C $f(x) = x^3 + 9x^2 + 20x$

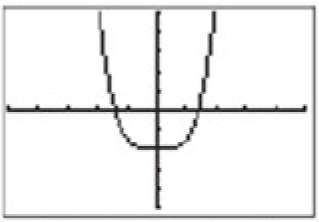
D $f(x) = x^4 - 5x^2 + 4$

SOLUTION:

In order for a function to have 3 turning points, the function has to have a degree of at least 4. This eliminates choice C.

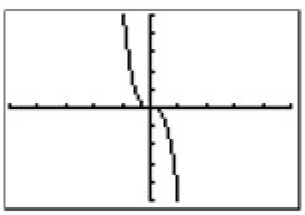
Use a graphing calculator to graph the remaining functions.

A



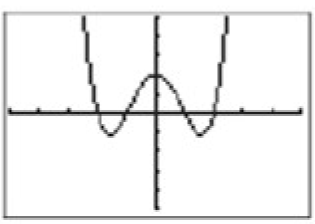
[−5, 5] scl: 1 by [−10, 10] scl: 2

B



[-5,5] scl: 1 by [-10,10] scl: 2

D



[−5, 5] scl: 1 by [−10, 10] scl: 2

The correct answer is D.

ANSWER:

D

For each function, (a) apply the leading term test, (b) find the zeros and state the multiplicity of any repeated zeros, (c) find a few additional points, and then (d) graph the function.

15. $f(x) = x(x - 1)(x + 3)$

SOLUTION:

a. The degree is 3, and the leading coefficient is 1. Because the degree is odd and the leading coefficient is positive, $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = \infty$.

b. The zeros are 0, 1, –3.

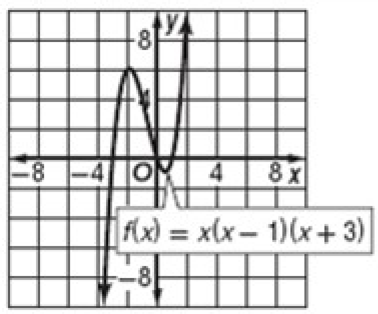
c. Sample answer: Evaluate the function for a few *x*-values in its domain.

x	−2	−1	2	3
***f*(*x*)**	6	4	10	36

d.
Evaluate the function for several *x*-values in its domain.

x	−5	−4	−3	0	1	4	5
***f*(*x*)**	−60	−20	0	0	0	84	160

Use these points to construct a graph.



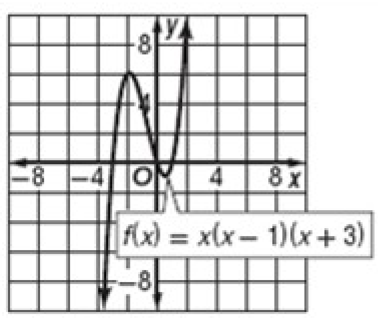
ANSWER:

a. The degree is 3 and the leading coefficient is 1. Because the degree is odd and the leading coefficient is positive, $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to-\infty} f(x) = -\infty$.

b. 0, 1, –3

c. Sample answer: (−1, 4), (−2, 6), (2, 10), (3, 36)

d.



16. $f(x) = x^4 - 9x^2$

SOLUTION:

a. The degree is 4, and the leading coefficient is 1. Because the degree is even and the leading coefficient is positive, $\lim_{x \to \infty} f(x) = \infty$ and $\lim_{x \to -\infty} f(x) = \infty$.

b.

$$0 = x^4 - 9x^2$$
$$0 = x^2(x^2 - 9)$$
$$0 = x^2(x - 3)(x + 3)$$

The zeros are 0, 3, and −3. The zero 0 has multiplicity 2 since x is a factor of the polynomial twice.

c. Sample answer: Evaluate the function for a few x-values in its domain.

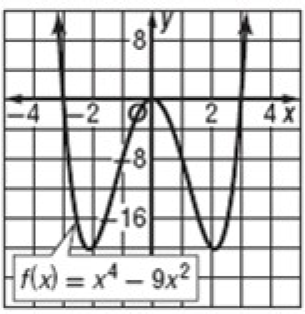
x	−1	1	2	4
***f* (*x*)**	−8	−8	−20	112

d.

Evaluate the function for several x-values in its domain.

x	−5	−4	−3	−2	0	3	5
***f* (*x*)**	400	112	0	−20	0	0	400

Use these points to construct a graph.



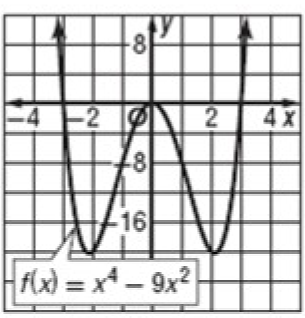
ANSWER:

a. The degree is 4 and the leading coefficient is 1. Because the degree is even and the leading coefficient is positive, $\lim_{x \to \infty} f(x) = \infty$ and $\lim_{x \to -\infty} f(x) = \infty$.

b. –3, 3, 0 (multiplicity 2)

c. Sample answer: (–1, –8), (1, –8), (2, –20), (4, 112)

d.



Use the Factor Theorem to determine if the binomials given are factors of $f(x)$. Use the binomials that are factors to write a factored form of $f(x)$.

17. $f(x) = x^3 - 3x^2 - 13x + 15$; $(x + 3)$

SOLUTION:

Use synthetic division to test the factor $(x + 3)$.

$$\begin{array}{r|rrrr} -3 & 1 & -3 & -13 & 15 \\ & & -3 & 18 & -15 \\ \hline & 1 & -6 & 5 & |0 \end{array}$$

Because the remainder when the polynomial is divided by $(x + 3)$ is 0, $(x + 3)$ is a factor of $f(x)$.

Because $(x + 3)$ is a factor of $f(x)$, we can use the final quotient to write a factored form of $f(x)$ as $f(x) = (x + 3)(x^2 - 6x + 5)$. Factoring the quadratic expression yields $f(x) = (x + 3)(x - 1)(x - 5)$.

ANSWER:

yes; $f(x) = (x + 3)(x - 1)(x - 5)$

18. $f(x) = x^4 - x^3 - 34x^2 + 4x + 120$; $(x + 5)$, $(x - 2)$

SOLUTION:

Use synthetic division to test each factor, $(x + 5)$ and $(x - 2)$.

$$\begin{array}{r|rrrrr} -5 & 1 & -1 & -34 & 4 & 120 \\ & & -5 & 30 & 20 & -120 \\ \hline & 1 & -6 & -4 & 24 & |0 \end{array}$$

Because the remainder when $f(x)$ is divided by $(x + 5)$ is 0, $(x + 5)$ is a factor. Test the second factor, $(x - 2)$, with the depressed polynomial $x^3 - 6x^2 - 4x + 24$.

$$\begin{array}{r|rrrr} 2 & 1 & -6 & -4 & 24 \\ & & 2 & -8 & -24 \\ \hline & 1 & -4 & -12 & |0 \end{array}$$

Because the remainder when the depressed polynomial is divided by $(x - 2)$ is 0, $(x - 2)$ is a factor of $f(x)$.

Because $(x + 5)$ and $(x - 2)$ are factors of $f(x)$, we can use the final quotient to write a factored form of $f(x)$ as $f(x) = (x + 5)(x - 2)(x^2 - 4x - 12)$. Factoring the quadratic expression yields $f(x) = (x + 5)(x - 2)(x - 6)(x + 2)$.

ANSWER:

yes; yes; $f(x) = (x - 2)(x + 5)(x - 6)(x + 2)$

Write a polynomial function of least degree with real coefficients in standard form that has the given zeros.

20. –1, 4, $\sqrt{3}$, $-\sqrt{3}$

SOLUTION:

Using the Linear Factorization Theorem and the zeros –1, 4, $\sqrt{3}$, and $-\sqrt{3}$, write $f(x)$ as follows.

$f(x) = a[x - (-1)][x - (4)][x - (\sqrt{3})][x - (-\sqrt{3})]$

Let a = 1. Then write the function in standard form.

$$\begin{aligned} f(x) &= (1)(x+1)(x-4)(x-\sqrt{3})(x+\sqrt{3}) \\ &= (x^2-3x-4)(x^2-3) \\ &= x^4-3x^3-7x^2+9x+12 \end{aligned}$$

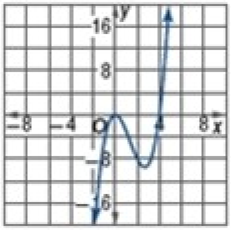
Therefore, a function of least degree that has 1, 4, $\sqrt{3}$, and $-\sqrt{3}$ as zeros is $f(x) = x^4 - 3x^3 - 7x^2 + 9x + 12$ or any nonzero multiple of $f(x)$.

ANSWER:

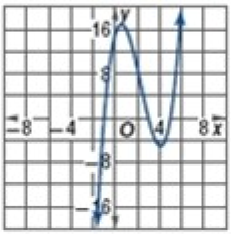
$f(x) = x^4 - 3x^3 - 7x^2 + 9x + 12$

22. **MULTIPLE CHOICE** Which function graphed below must have imaginary zeros?

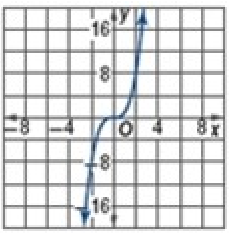
F



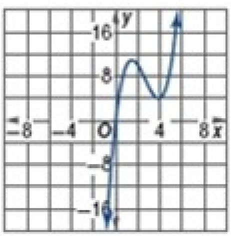
G



H



J



SOLUTION:

Analyze the turning points and real zeros of each graph. The graph for choice F has two turning points. Therefore, it must have at least 3 zeros. It appears to have two zeros, one of which having a multiplicity 2. Thus, the graph for choice F does not need to have any imaginary zeros.
The graph for choice G has two turning points. Therefore, it must have at least 3 zeros. The graph appears to have three zeros. Thus, the graph for choice G does not need to have any imaginary zeros.
The graph for choice H has zero turning points. It also has one zero. Thus, the graph for choice H does not need to have any imaginary zeros.
The graph for choice J has two turning points. Therefore, it must have at least 3 zeros. The graph appears to only have one zero. Thus, the graph for choice J must have at least two imaginary zeros.
The correct answer is J.

ANSWER:

J

Divide using synthetic division.

23. $f(x) = x^3 - 7x^2 + 13 \div (x - 2)$

SOLUTION:

Because $x - 2$, $c = 2$. Set up the synthetic division as follows. Then follow the synthetic division procedure.

$$\begin{array}{r|rrrr} 2 & 1 & -7 & 0 & 13 \\ & & 2 & -10 & -20 \\ \hline & 1 & -5 & -10 & -7 \end{array}$$

The quotient is $x^2 - 5x - 10 - \frac{7}{x-2}$.

ANSWER:

$x^2 - 5x - 10 - \frac{7}{x-2}$

24. $f(x) = x^4 + x^3 - 2x^2 + 3x + 8 \div (x + 3)$

SOLUTION:

Because $x + 3$, $c = -3$. Set up the synthetic division as follows. Then follow the synthetic division procedure.

$$\begin{array}{r|rrrrr} -3 & 1 & 1 & -2 & 3 & 8 \\ & & -3 & 6 & -12 & 27 \\ \hline & 1 & -2 & 4 & -9 & |35 \end{array}$$

The quotient is $x^3 - 2x^2 + 4x - 9 + \frac{35}{x+3}$.

ANSWER:

$x^3 - 2x^2 + 4x - 9 + \frac{35}{x+3}$

Practice Test - Chapter 2

Determine any asymptotes and intercepts. Then graph the function and state its domain.

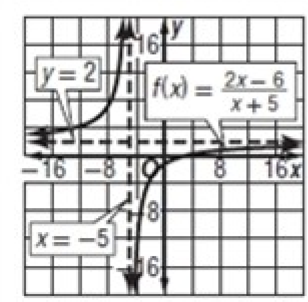
25. $f(x) = \frac{2x6}{x+5}$

SOLUTION:

The function is undefined at $b(x) = 0$, so D = $\{x \mid x \neq -5, x \in \mathbb{R}\}$.
There are vertical asymptotes at $x = -5$.
There is a horizontal asymptote at $y = 2$, the ratio of the leading coefficients of the numerator and denominator, because the degrees of the polynomials are equal.

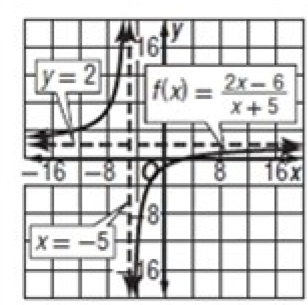
The x-intercept is 3, the zero of the numerator. The y-intercept is $-\frac{6}{5}$ because $f(0) = -\frac{6}{5}$.

Graph the asymptotes and intercepts. Then find and plot points.



ANSWER:

asymptotes: $x = -5$, $y = 2$; x-intercept: 3; y-intercept: $-\frac{6}{5}$; D = $\{x \mid x \neq -5, x \in \mathbb{R}\}$



26. $f(x) = \dfrac{x2 + x6}{x4}$

SOLUTION:

$f(x)$ can be written as $f(x) = \dfrac{(x-2)(x+3)}{x-4}$.

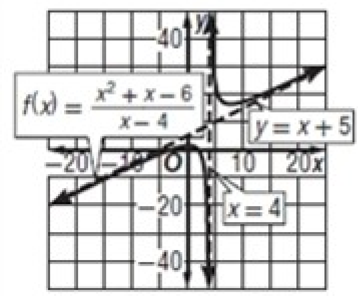
The function is undefined at $b(x) = 0$, so D = $\{x \mid x \neq 4, x \in \mathbb{R}\}$.
There are vertical asymptotes at $x = 4$.
Since the degree of the numerator is one larger than the degree of the denominator, the graph will have an oblique asymptote. Perform synthetic division to find the equation for the oblique asymptote.
Because $x - 4$, $c = 4$. Set up the synthetic division as follows. Then follow the synthetic division procedure.

$$\begin{array}{r|rrr} 4 & 1 & 1 & -6 \\ & & 4 & 20 \\ \hline & 1 & 5 & |14 \end{array}$$

The equation for the oblique asymptote is $y = x + 5$.

The x-intercepts are 2 and -3, the zeros of the numerator. The y-intercept is $\frac{3}{2}$ because $f(0) = \frac{3}{2}$.

Graph the asymptotes and intercepts. Then find and plot points.



ANSWER:

asymptotes: $x = 4$; $y = x + 5$; x-intercepts: –3 and 2; y-intercept: $\frac{3}{2}$; D = $\{x \mid x \neq 4, x \in \mathbb{R}\}$