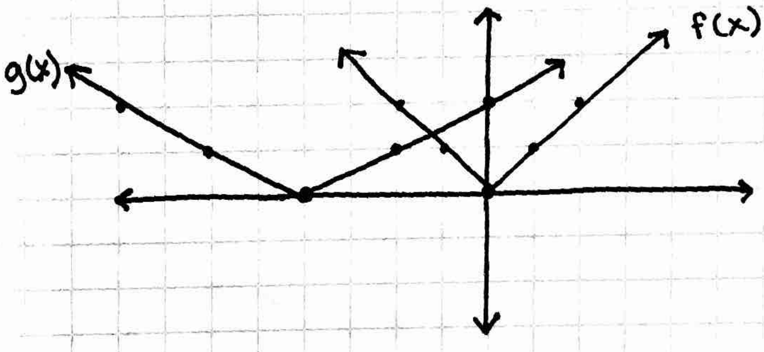
Worksheet

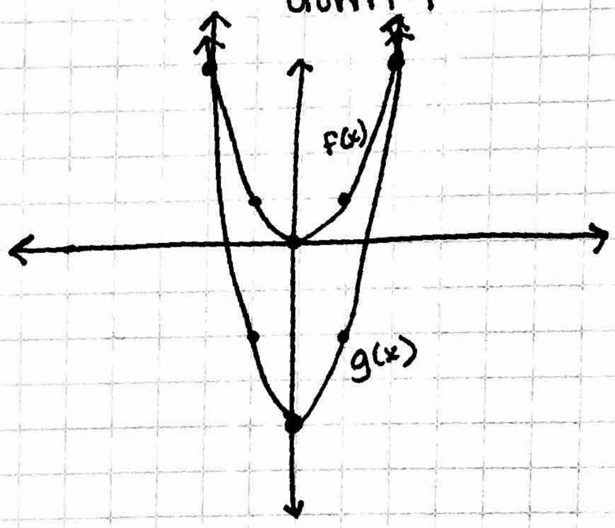
1. a) $g(x) = 0.5|x+4|$

PF is $f(x) = |x|$
relation: V.D. BAFO $1/2$
left 4

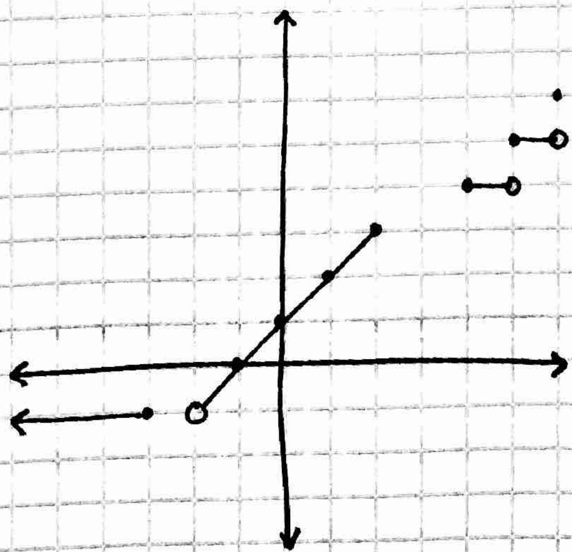


b) $g(x) = 2x^2 - 4$

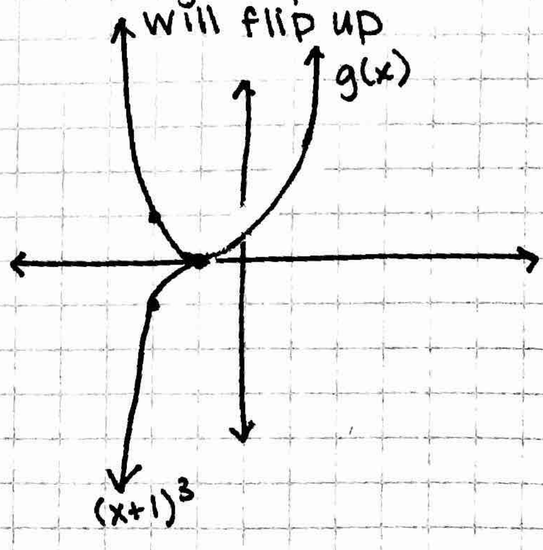
PF is $f(x) = x^2$
relation: V.D. BAFO 2
down 4



2. $f(x) = \begin{cases} -1 & \text{if } x \leq -3 \\ 1+x & \text{if } -2 < x \leq 2 \\ [[x]] & \text{if } 4 \leq x \leq 6 \end{cases}$



3. $g(x) = |(x+1)^3|$
everything below x-axis
will flip up



4. $f(g(x))=x$ & $g(f(x))=x$

a) $f(x)=2x+3$
$g(x)=\frac{x-3}{2}$

$f(g(x))=\cancel{2}\left(\frac{x-3}{\cancel{2}}\right)+3$
$f(g(x))=x-3+3$
$f(g(x))=x$ ✓

$g(f(x))=\frac{(2x+3)-3}{2}$

$g(f(x))=\frac{2x}{2}$

$g(f(x))=x$ ✓

yes, inverses

b) $f(x)=\frac{x^2}{2}-6 \quad x\geq 0$

$g(x)=\sqrt{2x+12}$

$f(g(x))=\frac{(\sqrt{2x+12})^2}{2}-6$

$f(g(x))=\frac{2x+12}{2}-6$

$f(g(x))=\frac{2x}{2}+\frac{12}{2}-6$

$f(g(x))=x+6-6=x$ ✓

$g(f(x))=\sqrt{\cancel{2}\left(\frac{x^2}{\cancel{2}}-6\right)+12}$

$g(f(x))=\sqrt{x^2-12+12}$

$g(f(x))=\sqrt{x^2}=x$ ✓

yes, inverses

5. a) $f(x)=2x^2+8$
$g(x)=5x-6$

$(f+g)(x)=2x^2+8+5x-6$
$(f+g)(x)=2x^2+5x+2 \quad D:(-\infty,\infty)$

$(f-g)(x)=2x^2+8-(5x-6)$
$(f-g)(x)=2x^2+8-5x+6$
$(f-g)(x)=2x^2-5x+14 \quad D:(-\infty,\infty)$

$(f\cdot g)(x)=(2x^2+8)(5x-6)$
$(f\cdot g)(x)=10x^3+40x-12x^2-48$
$(f\cdot g)(x)=10x^3-12x^2+40x-48 \quad D:(-\infty,\infty)$

$\left(\frac{f}{g}\right)(x)=\frac{2x^2+8}{5x-6} \quad D:(-\infty,6/5)\cup(6/5,\infty)$

5b) $f(x)=x^3$
$g(x)=\sqrt{x+1}$

$(f+g)(x)=x^3+\sqrt{x+1} \quad D:[-1,\infty)$

$(f-g)(x)=x^3-\sqrt{x+1} \quad D:[-1,\infty)$

$(f\cdot g)(x)=x^3\sqrt{x+1} \quad D:[-1,\infty)$

$\left(\frac{f}{g}\right)(x)=\frac{x^3}{\sqrt{x-1}} \quad D:(-1,\infty)$

6. a) $f(x) = 2x^2 - 5x + 1$
$g(x) = 2x - 3$

$(f \circ g)(x) = 2(2x-3)^2 - 5(2x-3) + 1$
$(f \circ g)(x) = 2(4x^2 - 12x + 9) - 10x + 15 + 1$
$(f \circ g)(x) = 8x^2 - 24x + 18 - 10x + 16$
$(f \circ g)(x) = 8x^2 - 34x + 34$

$(g \circ f)(x) = 2(2x^2 - 5x + 1) - 3$
$(g \circ f)(x) = 4x^2 - 10x + 2 - 3$
$(g \circ f)(x) = 4x^2 - 10x - 1$

$(f \circ g)(3) = 8(3)^2 - 34(3) + 34 = 4$

b) $f(x) = 2x^3 - 3x^2 + 1$
$g(x) = 3x$

$(f \circ g)(x) = 2(3x)^3 - 3(3x)^2 + 1$
$(f \circ g)(x) = 2(27x^3) - 3(9x^2) + 1$
$(f \circ g)(x) = 54x^3 - 27x^2 + 1$

$(g \circ f)(x) = 3(2x^3 - 3x^2 + 1)$
$(g \circ f)(x) = 6x^3 - 9x^2 + 3$

$(f \circ g)(3) = 54(3)^3 - 27(3^2) + 1 = 1216$

7. a) yes b) yes

8. a) $f(x) = \sqrt[3]{x-1}$ D: $(-\infty, \infty)$ R: $(-\infty, \infty)$

$y = \sqrt[3]{x-1}$
$(x)^3 = (\sqrt[3]{y-1})^3$

$x^3 = y - 1$
$x^3 + 1 = y$
$\boxed{f^{-1}(x) = x^3 + 1}$

D: $(-\infty, \infty)$

b) $f(x) = \dfrac{2x-1}{x+7}$ D: $(-\infty, -7) \cup (-7, \infty)$ R: $(-\infty, \infty)$

$y = \dfrac{2x-1}{x+7}$

$y+7 \cdot x = \dfrac{2y-1}{y+7} \cdot y+7$

$xy + 7x = 2y - 1$
$xy - 2y = -7x - 1$
$\dfrac{y\cancel{(x-2)}}{\cancel{x-2}} = \dfrac{-7x-1}{x-2}$

$y = \dfrac{-7x-1}{x-2}$ D: $(-\infty, 2) \cup (2, \infty)$

8. c) $\frac{4}{(x-3)^2} = f(x)$

no inverse

d) $\sqrt{x-2} = f(x)$ D: $[2, \infty)$ R: $[0, \infty)$

$y = \sqrt{x-2}$

$(x)^2 = (\sqrt{y-2})^2$

$x^2 = y-2$

$x^2+2 = y$

$f^{-1}(x) = x^2+2 \quad x \geq 0$

9. PF: x^2

transform: Down 5

Left 2

VD BAFO 12/49

$y = a(x+2)^2-5$

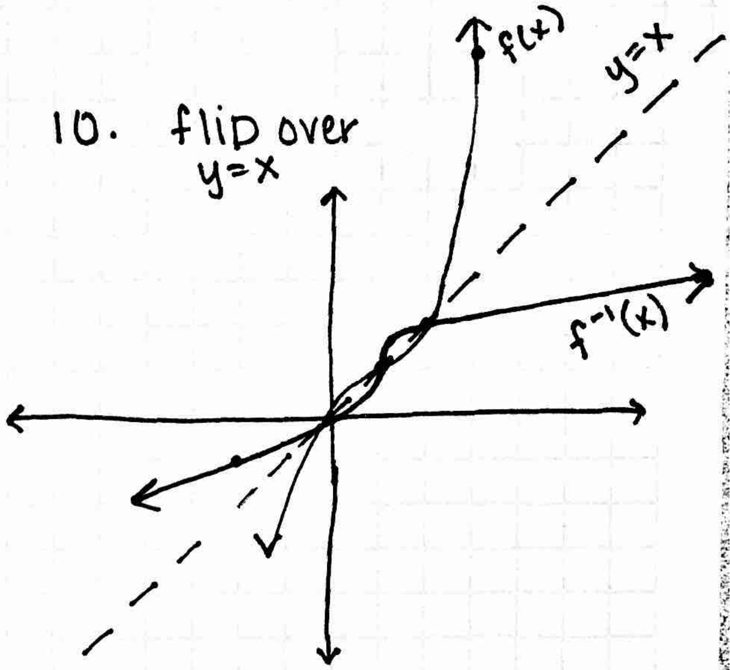
pt: (5,7) $7 = a(5+2)^2-5$

$12 = 49a$

$12/49 = a$

$$\boxed{f(x) = \frac{12}{49}(x+2)-5}$$

10. flip over $y=x$



11. $t(h) = \frac{\sqrt{h}}{4}$ D: $[0, \infty)$ R: $[0, \infty)$

$y = \frac{\sqrt{h}}{4}$

$4 \cdot h = \frac{\sqrt{y}}{4} \cdot 4$

$(4h)^2 = (\sqrt{y})^2$

$16h^2 = y$

$t^{-1}(h) = 16h^2 \quad h \geq 0$

since it is the inverse: plug in $8 = h$

$t^{-1}(8) = 16(8^2) = 1024$ ft