**Variable Growth Rate Modeling Activity**

We will examine a model that assumes *growth rate decreases as population size increases*.

**The Model**

We will use ***x*** to represent the **proportion of the population that is currently sick**. In this case, *x* 1 represents a population with 100% of the population sick, and 0 *x* 1 represents a population with both sick and healthy individuals.

* *x(t* +1*)* = proportion of infected people at time *t* 1
* *x(t)*, the proportion of people currently infected;
* 1*x*(*t* ), the proportion of people not infected;
* *r*, a growth factor that takes into account both the likelihood of interaction between people and the harmfulness of the disease.

To get a sense of how the dynamic plays out, we’ll create a table, by hand, with the first 10 iterates (or steps) with a **population of 600 people and 24 people sick initially**.

*x*00.04 , *r* 1.5 .

*x**t* 1*rx**t* 1*x*(*t* )  
   
 x(1) = 1.5(0.04)(1-0.04)

x(1) = 0.0576 (The % of people infected after 1 day (or 1 iteration)

x(2) = 1.5(0.0576)(1-0.0576) = 0.08142

**Q1. Use a calculator or Excel spreadsheet (instructions below) to fill the values in the table. Remember that the output from one step becomes the input for the next step.**

|  |  |  |
| --- | --- | --- |
| Iteration | Input (Percent Sick) | Output (percent Sick) |
| 1 | 0.04 | 0.0576 |
| 2 | 0.0576 | 0.08142 |
| 3 | 0.08142 |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |

**Using Excel to calculate percentage of sick people**

Open new spreadsheet

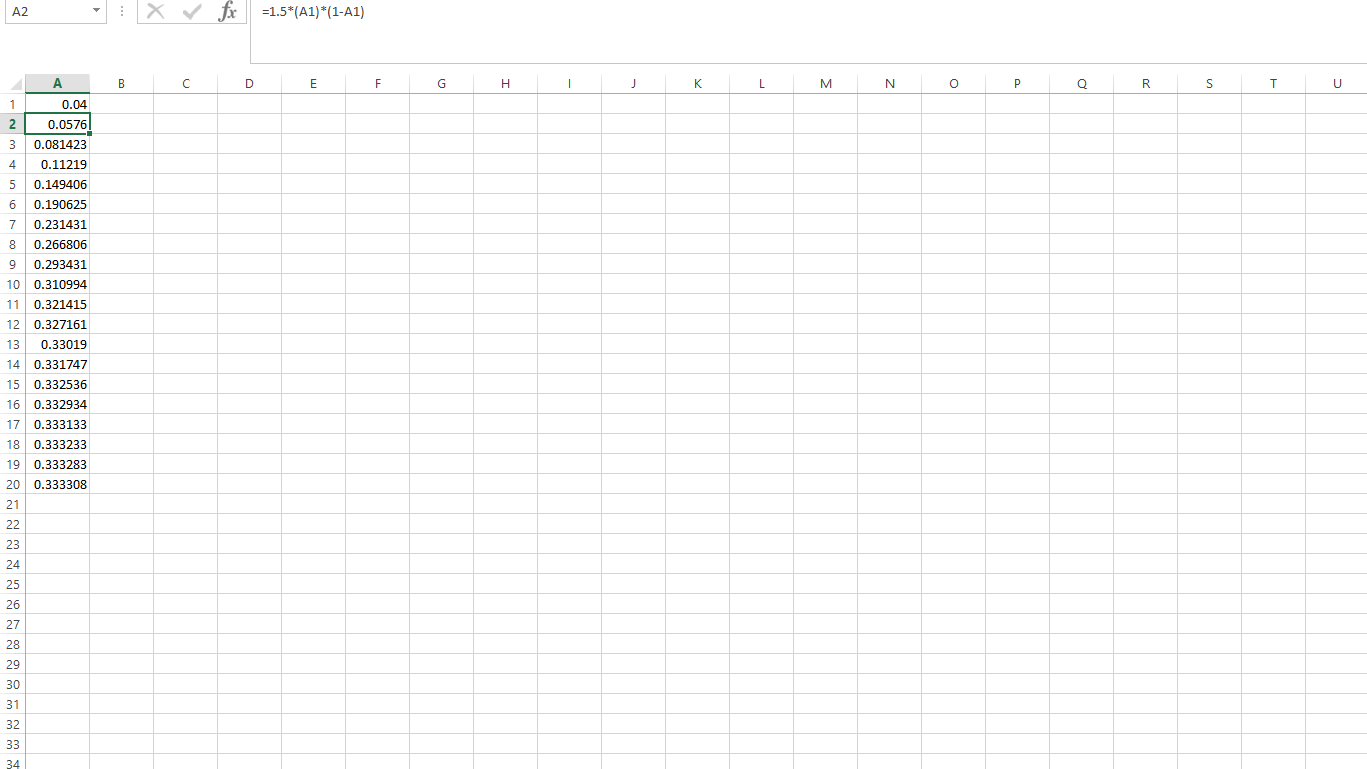
Click on cell A1 and type in 0.04

Click on cell A2 and type in =1.5\*(A1)\*(1-A1) then press ‘Enter’

Click on cell A2 again and copy cell by pressing ‘Ctrl + c’

Click on cell A3, hold ‘Shift’ key down and at the same time use down arrow key to highlight cells through A20

Press ‘Ctrl + v’ and you should see the screen below:



**Q2**. **What proportion of the population is sick after the 5th iteration?**

**How many students is this?**

**Graphing model using online tool**

Now create an Input-Output plot using online graphing tool.

1) Go to http://mathinsight.org/applet/function\_iteration\_cobweb\_combined

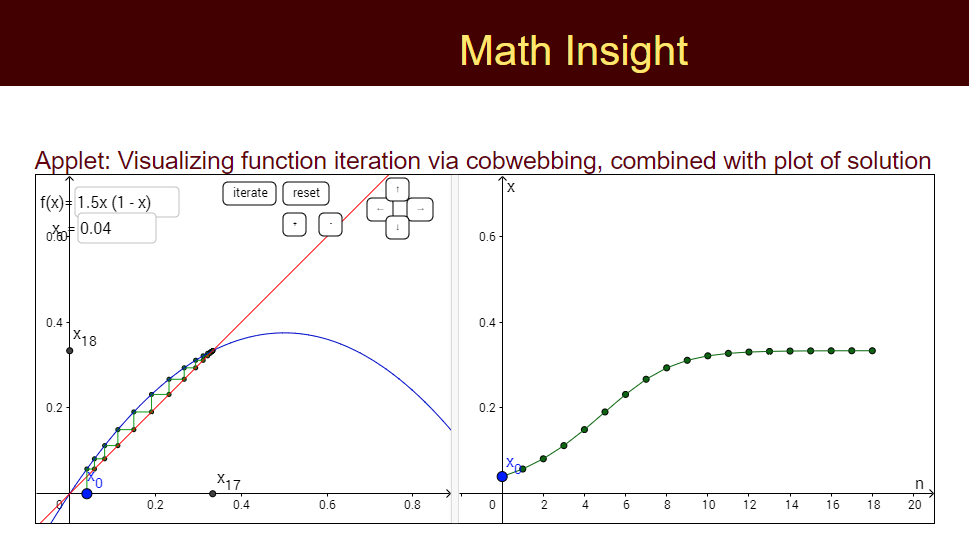
2) For f(x) type in 1.5(x)(1-x)

3) For x₀ type in 0.04   
4) Re-size window using the + and the arrow keys so each axis goes 0 to 0.6 or 0.8.

5) Click on the ‘Iterate’ button to generate plots.

7) Press reset for new graph using x₀= 0.5

The left panel shows the input-output while the right panel shows a plot of the results against the iteration number. The red line is the diagonal and the point where our graph intersects the diagonal is the *equilibrium or where the iteration reaches stabilization*.



**y= 1.5x(1-x)**

**y=x**

**Q3. Describe what is happening to the graphs. Are they stabilizing? If so, at what value?**

**Q4. How would you solve for the stabilization point (intersection of 2 graphs) algebraically?**

**Q5. What happens if we have a disease with a higher *r* value, indicating a greater likelihood of transmission? Use r value of 2.5 in the graphing tool: f(x)=2.5(x)(1-x)  
What is the stabilization point?**