5.1 Day 1 Trigonometric Identities   ex 1-3

Goal: Be able to use the reciprocal & pythagorean identities to solve problem

Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \qquad \cos x = \frac{1}{\sec x} \qquad \tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

Quotient Identities

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

ex) If $\sin x = \frac{1}{4}$ and $\cos x = \frac{2}{5}$ find $\cot x$

$$\cot x = \frac{\frac{2}{5}}{\frac{1}{4}} \qquad \boxed{\cot x = \frac{8}{5}}$$

y+) if $\sec x = \sqrt{5}$ + $\tan x = 2\sqrt{3}$ find $\sin x$

$$\cos x = \frac{1}{\sqrt{5}}$$

$$\cos x = \frac{\sqrt{5}}{5}$$

$$\left(\frac{\sqrt{5}}{5}\right) \frac{\sin x}{\frac{\sqrt{5}}{5}} = 2\sqrt{3}\left(\frac{\sqrt{5}}{5}\right)$$

$$\sin x = \frac{2\sqrt{15}}{5}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \cot^2 x = \csc^2 x$$

$$\tan^2 x + 1 = \sec^2 x$$

__ex)__ if $\cot x = 2$ and $\cos x < 0$ find $\sin x + \cos x$

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$$1 + (2)^2 = \csc^2 x$$

$$5 = \csc^2 x$$

$$\pm \sqrt{5} = \csc x$$

$$-\sqrt{5} = \csc x$$

$$\frac{-1}{\sqrt{5}} = \sin x$$

$$\boxed{-\frac{\sqrt{5}}{5} = \sin x}$$

$$\left(-\frac{\sqrt{5}}{5}\right) 2 = \left(\frac{\cos x}{\frac{-\sqrt{5}}{5}}\right) -\frac{\sqrt{5}}{5}$$

$$\boxed{-\frac{2\sqrt{5}}{5} = \cos x}$$

$$\left(-\frac{\sqrt{5}}{5}\right)^2 + \cos^2 x = 1$$

$$\frac{5}{25} + \cos^2 x = 1$$

$$\cos^2 x = \frac{20}{25}$$

$$\cos^2 x = \frac{4}{5}$$

$$\cos x = \pm\sqrt{\frac{4}{5}}$$

$$= -\frac{2}{\sqrt{5}} = \frac{-2\sqrt{5}}{5}$$

__y)__ $\cot x = -3 \qquad \cos x > 0 \qquad$ find $\csc x$

$$1 + (-3)^2 = \csc^2 x$$

$$10 = \csc^2 x$$

$$\pm\sqrt{10} = \csc x$$

$$-\sqrt{10} = \csc x$$